NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIALS

MET201 MECHANICS OF SOLIDS

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- Established in: 2002
- Course offered : B.Tech in Mechanical Engineering

- ♦ Approved by AICTE New Delhi and Accredited by NAAC
- ♦ Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

DEPARTMENT VISION

Producing internationallycompetitive Mechanical Engineerswith social responsibility & sustainable employability through viable strategies as well as competent exposure oriented quality education.

DEPARTMENT MISSION

1. Imparting high impacteducation by providing conductive teaching learning environment.
2. Fostering effective modes of continuous learning process with moral & ethical values.
3. Enhancing leadership qualities with social commitment, professional attitude, unity, team spirit & communication skill.
4. Introducing the present scenario in research & development through collaborative efforts blended with industry & institution.

PROGRAMME EDUCATIONAL OBJECTIVES

PEO1: Graduates shall have strong practical & technical exposures in the field of Mechanical Engineering & will contribute to the society through innovation & enterprise.

PEO2: Graduates will have the demonstrated ability toanalyze, formulate & solve design engineering / thermal engineering / materials & manufacturing / design issues & real life problems.

PEO3: Graduates will be capable of pursuing Mechanical Engineering profession with good communication skills, leadership qualities, team spirit & communication skills.

PEO4: Graduates will sustain an appetite for continuous learning by pursuing higher education & research in the allied areas of technology.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: graduates able to apply principles of engineering, basic sciences & analytics including multi variant calculus & higher order partial differential equations..

PSO2: Graduates able to perform modeling, analyzing, designing & simulating physical systems, components & processes.

PSO3: Graduates able to work professionally on mechanical systems, thermal systems & production systems.

MAPPING OF COURSE OUTCOMES WITH PROGRAM SPECIFIC OUTCOMES

	PSO1	PSO2	PSO3
CO1	3	3	2
CO2	3	3	2
CO3	2	3	1
CO4	3	2	2
CO5	2	3	2

Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

SYLLABUS

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MET201	MECHANICS OF SOLIDS	PCC	3	1	0	4

Preamble:

This course helps the students to understand the concept of stress and strain in different types of structure/machine under various loading conditions. The course also covers simple and compound stresses due to forces, stresses and deflection in beams due to bending, torsion in circular section, strain energy, different theories of failure, stress in thin cylinder thick cylinder and spheres due to external and internal pressure.

Prerequisite: EST100 ENGINEERING MECHANICS

Course Outcomes:

After the completion of the course the student will be able to

CO 1	Determine the stresses, strains and displacements of structures by tensorial and graphical (Mohr's circle) approaches
CO 2	Analyse the strength of materials using stress-strain relationships for structural and thermal loading
CO 3	Perform basic design of shafts subjected to torsional loading and analyse beams subjected to bending moments
CO 4	Determine the deformation of structures subjected to various loading conditions using strain energy methods
CO 5	Analyse column buckling and appreciate the theories of failures and its relevance in engineering design

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	2									1
CO 2	3	3	2									1
CO 3	3	3	1									2
CO 4	3	3	1									1
CO 5	3	3	1									1

Assessment Pattern

Bloom's Category	Continuous Assessment Tests		End Semester Examination
	1	2	
Remember	10	10	20
Understand	20	20	30
Apply	20	20	50
Analyse			
Evaluate			
Create			

Mark distribution

Total Marks	CIE	ESE	ESE Duration
150	50	100	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 10 marks
Continuous Assessment Test (2 numbers) : 25 marks
Assignment/Quiz/Course project : 15 marks

End Semester Examination Pattern:

There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module and having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question carries 14 marks and can have a maximum of 2 subdivisions.

COURSE LEVEL ASSESSMENT QUESTIONS

Course Outcome 1 (CO1):

1. Determine the resultant traction at a point in a plane using the stress tensor.

2. Evaluate the principal stresses, principal strains and their directions from a given state of stress or strain.

3. Write the stress tensor and strain tensor.

Course Outcome 2 (CO2)

1. Write the generalized Hooke's law for stress-strain relations.

2. Estimate the state of strain from a given state of stress.

3. Analyse the strength of a structure subjected to thermal loading.

Course Outcome 3(CO3):

1. Design a shaft to transmit power and torque.

2. Draw the shear force and bending moment diagrams.

3. Determine the bending stress on a beam subjected to pure bending.

Course Outcome 4 (CO4):

1. Apply strain energy method to estimate the deformation of a structure.

2. Use strain energy method to calculate deformations for multiple loads.

3. Use strain energy method to estimate the loads acting on a structure for a maximum deflection.

Course Outcome 5 (CO5):

1. Analyse a column for buckling load.

2. Use Rankine formula to determine the crippling load of columns.

3. A bolt is subjected to a direct tensile load of 20 kN and a shear load of 15 kN. Suggest suitable size of this bolt according to various theories of elastic failure, if the yield stress in simple tension is 360 MPa. A factor of safety 2 should be used. Assume Poisson's ratio as 0.3.

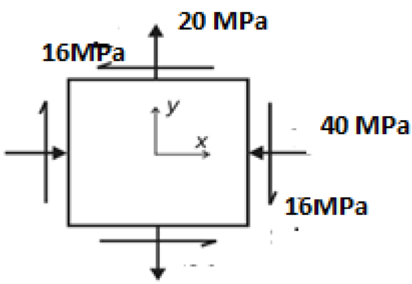
COURSE PLAN

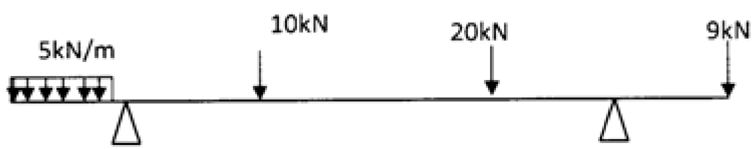
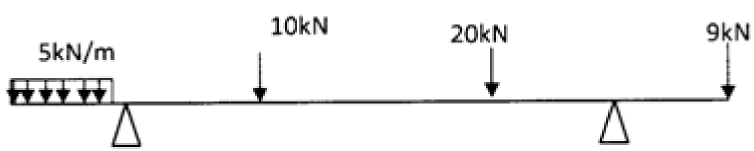
No	Topic	No of lectures
1	**Module 1: Stress and Strain Analysis**	**9 hours**
1.1	Describe the deformation behaviour of elastic solids in equilibrium under the action of a system of forces. Describe method of sections to illustrate stress as resisting force per unit area. Stress vectors on Cartesian coordinate planes passing through a point and writing stress at a point in the form of a matrix.	2 hr
1.2	Equality of cross shear (Derivation not required). Write Cauchy's equation (Derivation not required),Find resultant stress, Normal and shear stress on a plane given stress tensor and direction cosines (no questions for finding direction cosines).	2 hr
1.3	Displacement, gradient of displacement, Cartesian strain matrix, Write strain-displacement relations (small-strain only), Simple problems to find strain matrix given displacement field (2D and 3D), write stress tensor and strain tensor for Plane stress and plane strain conditions.	1 hr
1.4	Concepts of principal planes and principal stress, characteristic equation of stress matrix and evaluation of principal stresses and principal planes as an eigen value problem, meaning of stress invariants, maximum shear stress	2 hrs
1.5	Mohr's circle for 2D case: find principal stress, planes, stress on an arbitrary plane, maximum shear stress graphically using Mohr's circle	2 hrs
2	**Module 2: Stress - Strain Relationships**	**9 hours**
2.1	Stress-strain diagram, Stress–Strain curves of Ductile and Brittle Materials, Poisson's ratio	1 hr
2.2	Constitutive equations-generalized Hooke's law, equations for linear elastic isotropic solids in in terms of Young's Modulus and Poisson's ratio (3D). Hooke's law for Plane stress and plane strain conditions Relations between elastic constants E, G, ν and K(derivation not required), Numerical problems	2 hrs
2.3	Calculation of stress, strain and change in length in axially loaded members with single and composite materials, Effects of thermal loading – thermal stress and thermal strain. Thermal stress on a prismatic bar held between fixed supports.	2 hrs
2.4	Numerical problems for axially loaded members	4 hrs
3	**Module 3: Torsion of circular shafts, Shear Force-Bending Moment Diagrams and Pure bending**	**9 hours**
3.1	Torsional deformation of circular shafts, assumptions for shafts subjected to torsion within elastic deformation range, derivation of torsion formula	1 hr
3.2	Torsional rigidity, Polar moment of inertia, comparison of solid and hollow shaft. Simple problems to estimate the stress in solid and hollow shafts	1 hr
3.3	Numerical problems for basic design of circular shafts subjected to externally applied torques	1 hr
3.4	Shear force and bending moment diagrams for cantilever and simply	2 hrs

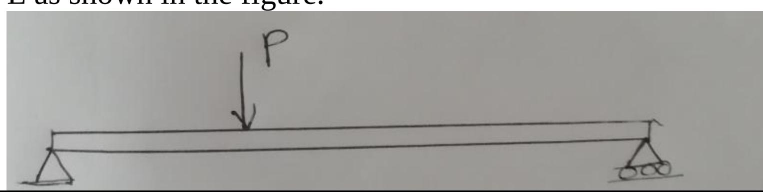
	supported beams subjected to point load, moment, UDL and linearly varying load	
3.5	Differential equations between load, shear force and bending moment.	1 hr
3.6	Normal and shear stress in beams: Derivation of flexural formula, section modulus, flexural rigidity, numerical problems to evaluate bending stress, economic sections Shear stress formula for beams: (Derivation not required),numerical problem to find shear stress distribution for rectangular section	3 hrs
4	**Module 4: Deflection of beams, Strain energy**	**8 hours**
4.1	Deflection of cantilever and simply supported beams subjected to point load, moment and UDL using Macauley's method (procedure and problems with multiple loads)	2 hrs
4.2	Linear elastic loading, elastic strain energy and Complementary strain energy. Elastic strain energy for axial loading, transverse shear, bending and torsional loads (short derivations in terms of loads and deflections).	2 hr
4.3	Expressions for strain energy in terms of load, geometry and material properties of the body for axial, shearing, bending and torsional loads. Simple problems to solve elastic deformations	2 hrs
4.4	Castigliano's second theorem to find displacements, reciprocal relation, (Proof not required for Castigliano's second theorem and reciprocal relation).	1 hr
4.5	Simple problems to find the deflections using Castigliano's theorem	1 hr
5	**Module 5: Buckling of Columns, Theories of Failure**	**8 hours**
5.1	Fundamentals of bucking and stability, critical load, Euler's formula for long columns, assumptions and limitations, effect of end conditions(derivation only for pinned ends), equivalent length	2 hr
5.2	Critical stress, slenderness ratio, Rankine's formula for short columns, Problems	3 hr
5.3	Introduction to Theories of Failure. Rankine's theory for maximum normal stress, Guest's theory for maximum shear stress, Saint-Venant's theory for maximum normal strain	2 hr
5.4	Hencky-von Mises theory for maximum distortion energy, Haigh's theory for maximum strain energy	1 hr

QUESTION BANK

MODULE I

Q:NO:	QUESTIONS	CO	KL
1	What you mean bystate of stress at a point?	CO1	K2
2	State of stress at a point is given by cartesian stress tensor $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 2 \end{bmatrix} kPa$ Find the following (a) Stress invariants (b)Characteristic equation (c)Principal stress	CO1	K4
3	Explain strain tensor.	CO1	K2
4	The state of plane stress at a point is represented by the stress element below . The stresses acting are σ_x =40 Mpa, σ_y= 20 Mpa and τ_{xy}= 16Mpa.Draw Mohr's circle , determine the principal stresses and Maximum shear stresses 	CO1	K4
5	The state of stress at a point is given by σ_x = 2kpa, σ_y =3 kPa, σ_z = 2kPa, τ_{xy} = - 1kPa,τ_{xz} =1kPa,τ_{yz} = -1kPa. Determine the principal stresses	CO1	K2
6	The state of stress at a point is given by σ_x = 70Mpa σ_y = 10Mpa σ_z = -20Mpa τ_{xy} = - 40MPa τ_{xz} = τ_{yz} = 20MPa. Determine the principal stresses	CO1	K2
7	Displacement field for a 2D plane strain is u = $[(x^2+xy)\ i + (y^2+xy)\ j] * 10^{-2}$, Find component of strain at a point (2,4) .	CO1	K2
8	The displacement field in a body is given by u = $[(x^2+y)\ i + (y+2)\ j + (x^2-2z^2)\ k\ ] * 10^{-3}$. Determine the principal strain at the point (2,2,3) and direction of maximum principal strain.	CO1	K4

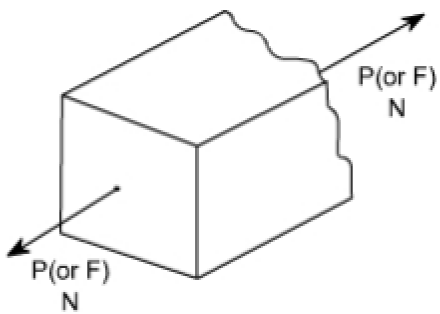
	MODULE II		
1	Explain Hooke's law for linearly elastic isotropic material.	CO2	K2
2	Write equations of generalised Hooks law of elasticity	CO2	K4
3	Write down relationship among elastic constant	CO2	K2
4	Explain stress-strain curve mild steel bar in tension test	CO2	K5
5	What you mean by principle of superposition	CO2	K2
6	Write equations of generalised Hooks law of elasticity.	CO2	K3
7	Write down Cauchy's strain displacement relationship.	CO2	K5
8	A composite bar made of brass and steel is fixed between two supports as shown in igure. If the temperature is increased by 80^0 C. Find the stresses induced in steel and brass section. Assuming (i)the support do not yied (ii) Support yied by 0.15mm. Take E_S= 200Gpa , E_B= 100Gpa, α_S= 12 *10^{-6} /^{0}C, α_B= 19 *10^{-6} /^{0}C, L_S= 200 mm, L_B= 250mm	CO2	K4
9	Derive the relationship between stress and strain for an isotropic materialin terms of Lame's Coefficient	CO2	K4
	MODULE III		
1	What you mean by torsion?	CO3	K3
2	Explainassumptions of torsional equation.	CO3	K3
3	Write a short note on torsional rigidity and fleural rigidity	CO3	K2
4	Draw the shear force and bending moment diagram for the beam loaded and supported as shown in figure 	CO3	K3
5	Draw shear force and bending moment digram for overhanging beam shown below 	CO3	K5
6	Explain section modulus.	CO3	K3
7	Compare the strength of a hollow shaft of diameter ratio 0.85 to that	CO3	K2

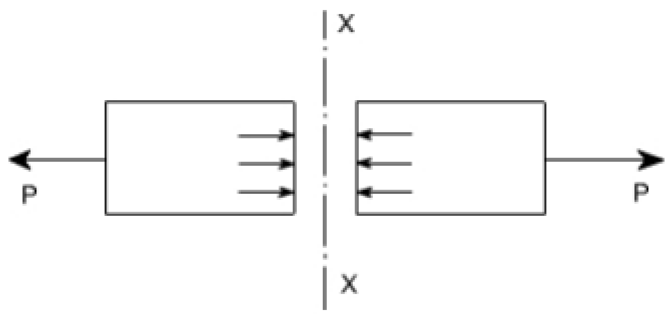
	of solid shaft by considering the permissible shear stress. Both the shaft are of same material, of same length and same weight.		
8	Compare the weight of a hollow shaft of diameter ratio 0.85 to that of solid shaft by considering the permissible shear stress. Both the shaft are of same material, of same length and same strength.	CO3	K5
9	Derive torsion equation	CO3	K5
10	Derive the epression for shear stress in a beam symmetrical and unsymmetrical section?	CO3	K4
	MODULE IV		
1	Write down reciprocal relation for multiple loads on a structure	CO4	K2
2	Derive the expressions for elastic strain energy in terms of applied load/moment and material property for the cases of a) Axial force b) Bending moment c)shear force and d)torque	CO4	K4
3	Calculate the displacement in the direction of load P applied at a distance of L/6 from the left end for a simply supported beam of span L as shown in the figure. 	CO4	K4
4	Define and proove Castigliano's second theorem	CO4	K2
5	A beam 6 m long is freely supported at its ends. It carries concentrated load of 50 kN each at points 2m from the ends . Calculate (a) Slope and Deflection under load (b) maximum deflection and slope of the beam Use Macaulay's method and take Fleural rigidity of the beam = 13000 kN m^2	CO4	K4
6	Derive bending equation	CO4	K4
	MODULE V		
1	Explain theories of failure	CO5	K2
2	Derive Euler's formula for a column with ond end hinged and other end fixed	CO5	K4
3	Derive rankine formula.	CO5	K4
4	Difference between column and struct	CO5	K2
5	A cylindrical shell 4m long closed at the ends has an internal diameter of 2m and wall thickness 16mm. Calculate circumferential and longitudinal stress induced and also the change in dimensions of the shell, if it is subjected to an internal pressure of 2.5Mpa. Take E= $28*10^5$ N/mm^2 and	CO5	K4

	poisson's ratio= 0.3		
6	Define critical load	CO5	K2
7	Find the crippling load for a hollow steel column 60mm internal diameter and 6mm thick. The column is 5m long with one end fixed and other end hinged. Use Rankine's formula and Rankine'sconstant as 1/6500 and σc = 330 N/mm^2. Compare this load by crippling load given by Euler's formula. Take E = 110 GPa.	CO5	K2

Module1

Stress Stress is the internal resistance offered by the body to the external load applied to it per unit cross sectional area. Stresses are normal to the plane to which they act and are tensile or compressive in nature.





As we know that in mechanics of deformable solids, externally applied forces acts on a body and body suffers a deformation. From equilibrium point of view, this action should be opposed or reacted by internal forces which are set up within the particles of material due to cohesion. These internal forces give rise to a concept of stress. Consider a rectangular rod subjected to axial pull P. Let us imagine that the same rectangular bar is assumed to be cut into two halves at section XX. The each portion of this rectangular bar is in equilibrium under the action of load P and the internal forces acting at the section XX has been shown. Now stress is defined as the force intensity or force per unit area. Here we use a to represent the stress.σsymbol

$$\sigma = \frac{P}{A}$$

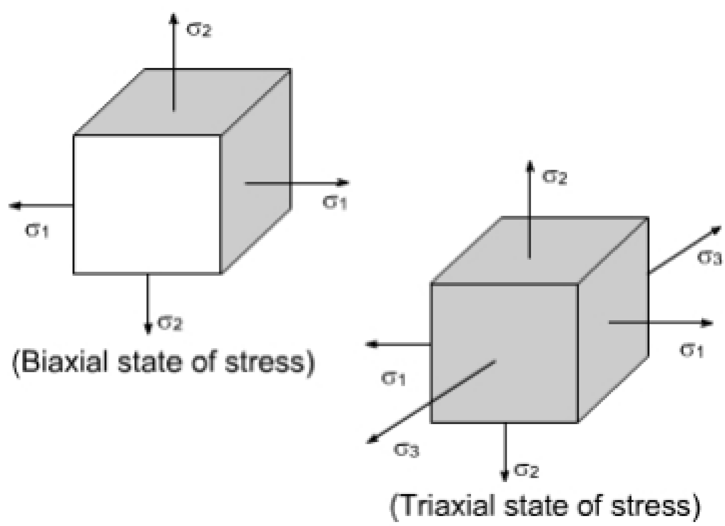
Here we are using an assumption that the total force or total load carried by the rectangular bar is uniformly distributed over its cross – section. But the stress distributions may be for from uniform, with local regions of high stress known as stress concentrations. If the force carried by a component is not uniformly distributed over its cross – sectional area, A, we must consider a small area, ‘δA’ which carries a small load ‘δP’, of the total force ‘P', Then definition of stress is

$$\boxed{\sigma = \frac{\delta F}{\delta A}}$$

Units : The basic units of stress in S.I units i.e. (International system) are N / m2 (or Pa) While US customary unit is pound per square inch psi.

TYPES OF STRESSES : Only two basic stresses exists : (1) normal stress and (2) shear stress. Other stresses either are similar to these basic stresses or are a combination of this e.g. bending stress is a combination tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress. Let us define the normal stresses and shear stresses in the following sections.

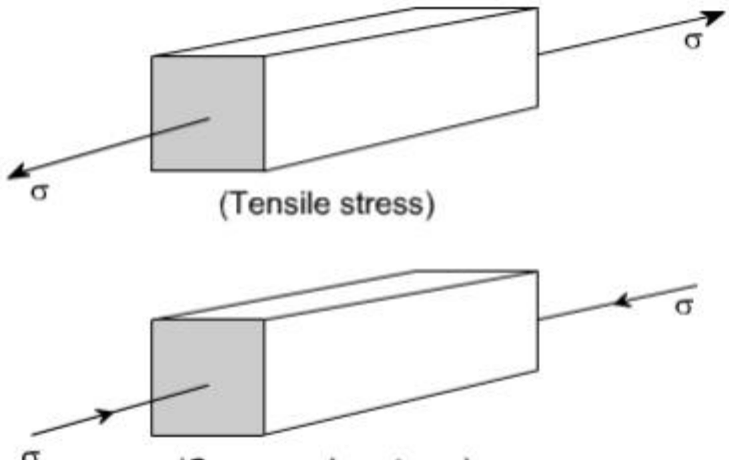
Normal stresses : We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter (σ)



(Biaxial state of stress)

(Triaxial state of stress)

Tensile or compressive Stresses: The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area

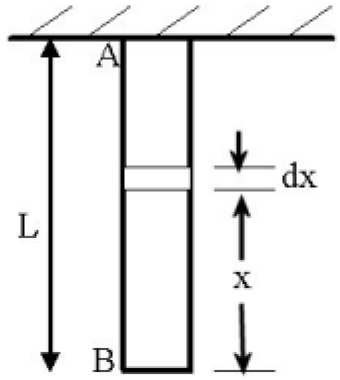


(Tensile stress)

(Compressive stress)

Shear Stresses: Let us consider now the situation, where the cross – sectional area of a block of material is subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting stress is known as shear stress.

Complementary shear stresses: The existence of shear stresses on any two sides of the element induces complementary shear stresses on the other two sides of the element to maintain in sides AB and CD induces aτequilibrium. As shown in the figure the shear stress in sides AD and BC.τcomplimentary shear stress '



Let

L= length of the bar

A= cross-sectional area of the bar

E= Young's modulus of the bar material

w= specific weight of the bar material

Then deformation due to the self-weight of the bar is $\boxed{\delta L = \dfrac{WL}{2E}}$

Mechanics of Solids (ME-201)

- Multidisciplinary subject applicable to all core branches of Engineering like, Mechanical, Civil, Automobile Engineering, Aerospace Engineering, Architecture.

- MOS forms the fundamental essential knowledge for engineers who have to work with materials, solids and deformable bodies.

- This course provides basic concepts in day to day engineering problems which involves materials, machines or structures/systems.

- The fundamental knowledge provided by MOS will form the base to another advanced levels of engineering design and future career or advanced courses like Post graduate/Research level.

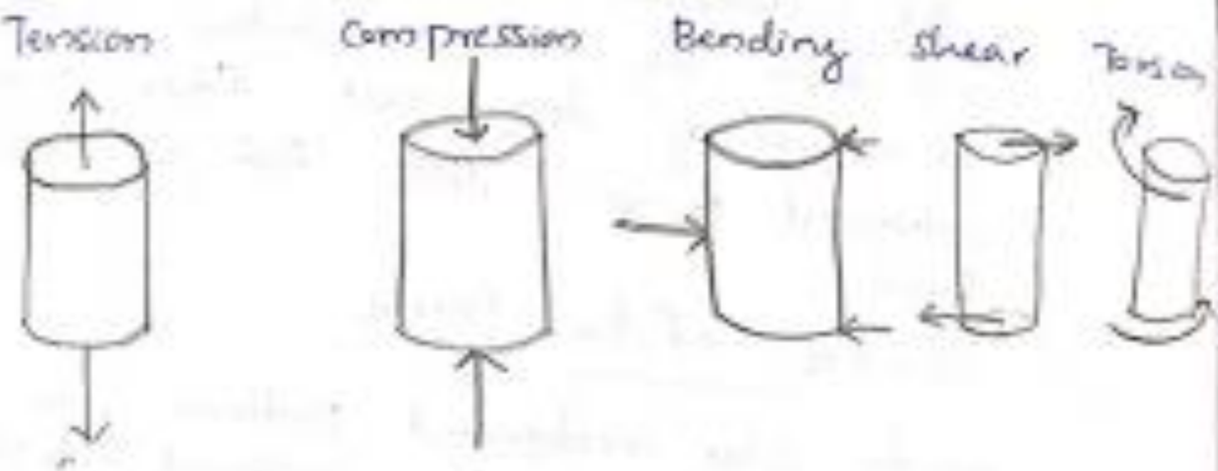
Objective of the course

- to solve mechanical problems
- to understand the material behaviour under loads
- to do the static analysis of component part of a system to find the internal actions, force or moment.
- to determine the stresses, strain and forces due to internal action (means the response of the structure to the action of external loads)
- to compare the stress/strain with acceptable value
- to improve the engineering design skills

Terms associated with the mechanics of deformable bodies

Type of loading

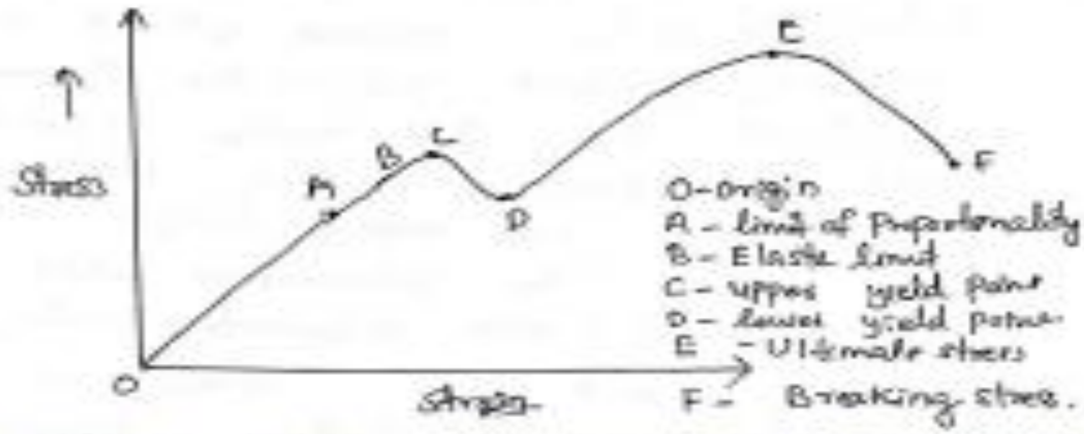
- The application of force to an object is called Loading
- The body undergoes deformation when it is subjected to loading.
- The mechanics of deformable Solids is more concerned with the internal forces and associated changes in the geometry of the components involved
- There are five basic fundamental loading conditions



Equilibrium of a body

The body is said to be in equilibrium, when the internal forces and moments acting on the body are in equilibrium.

Stress Strain Curve of an elastic material - Mild Steel



The above figure represents the stress - strain of mild steel obtained from uniaxial tension test. A specimen of standard shape and size is used for conducting uniaxial tension test, specimen is mounted on the Universal Testing machine (UTM), using suitable grips at the gear guage length marked on the specimen. Specimen is then subjected to slowly and steadily applied tensile loading. The elongation δL over the specified gauge length is determined by a measuring device called extensometer.

A - Proportionality Limit - It is the stress upto which stress - strain diagram is to be a straight line.

B - Elastic limit - It is the maximum stress developed on the tension side of a specimen such that there is no permanent deformation

C- Yield point

It is the point at which appreciable elongation of the specimen without any corresponding increase in load. This is the phenomena of structural steel, that makes it suitable for construction purpose. Here a smaller value of applied stress can develop larger strain on the material. This phenomenon which occurs on the onset of plastic deformation, also known as yielding and the stress at which yielding occurs first is known as upper yield point.

D- Lower yield point

After point C, the curve dip down slightly and again shifts upwards. The point D represent the stage where further increase in loading results in the neck formation and the failure of specimen.

E- Ultimate point

It is the stress corresponding to the failure of the specimen, as a result of neck formation and therefore it is known as rupture strength.

Due to the neck formation the cross sectional area of the test specimen reduces considerably and the actual rupture strength is obtained by dividing the breaking load by the

C/s area at the time of rupture

Hooke's law

Most of the engineering materials show elastic behaviour only up to a certain limit.

Hooke's law states that with in elastic limit

Stress ∝ strain

$\sigma \propto \epsilon$

$\frac{\sigma}{\epsilon}$ = a constant, E , where E = modulus of elasticity/ Young's modulus.

unit is N/mm² or Mpa.

Modulus of Rigidity

For elastic materials shear stress is proportional to shear stress with in elastic limit.

The Ratio of shear stress to shear strain is known as modulus of Rigidity. denoted by 'G'

$$G = \frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\phi)}$$

$$G = \frac{\tau}{\phi}.$$

Elongation of a uniform bar

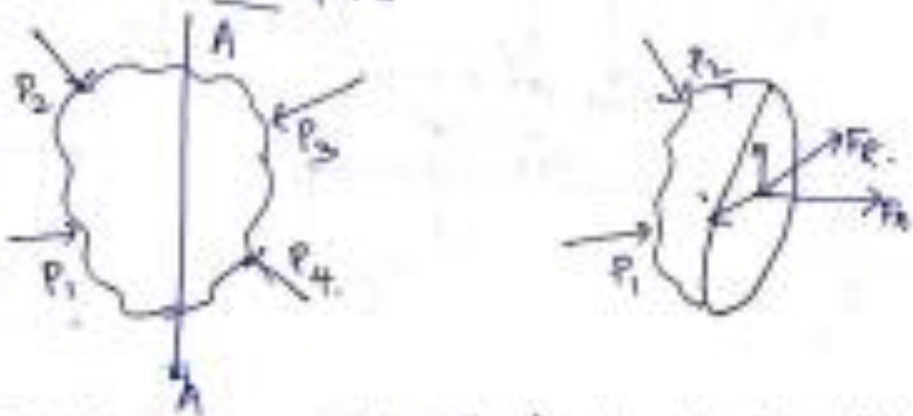
Young's modulus, E $= \frac{\text{stress}}{\text{strain}}.$

$$= \frac{P/A}{\delta l/l}.$$

$$\frac{\delta l}{l} = \frac{P}{AE}$$

3 Dimensional State of stress

Stress at a point

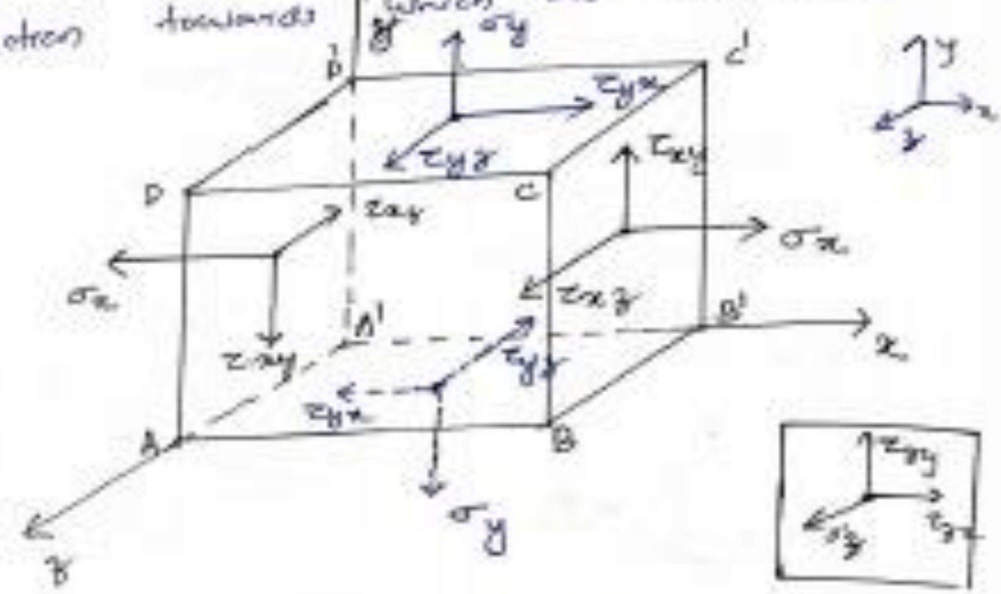


Consider an arbitrary body in equilibrium under the action of a set of forces. Consider section A-A which divides the area body into two halves.

If you consider one such part, there will be a net force acting at the cross section to keep the body in equillibrium. This force can be resolved in three mutually perpendicular directions. One normal to the plane AA and other two tangential to the plane. The resolved components divided by the area A-A gives normal stress and shear stress respectively.

Double subscript system is used to represent the stress at a point. The first subscript denotes the direction of the outward drawn normal

plane on which ...
on the ... subscript denotes the
and the second subscript denotes the stress acts.
direction towards which the stress acts.



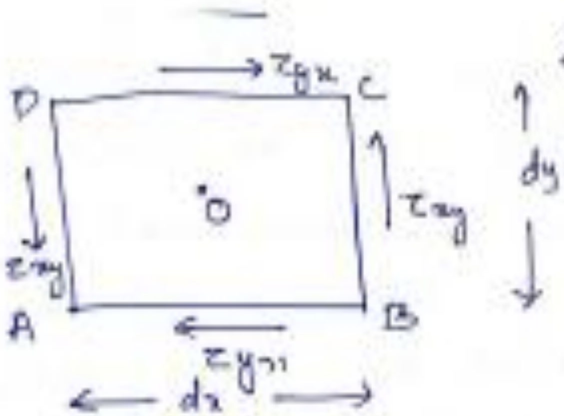
Hence an area perpendicular to the x axis stresses are written as τ_{xx}, τ_{xy}, τ_{xz}.
τ_{xx} is written as σ_x, which a stress normal to the area.
When we represent the 9 components of stress in a matrix known as stress tensor.

$$\tau_{ij} = \begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} x \\ y \\ z \end{array} & \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \end{array}$$

→ Stress tensor matrix in Cartesian co-ordinate

Scalar quantity T, P (1)
Vector v, a (3 v_x, v_y, v_z)
Tensor (9 nos).

from previous figure.



Considering the equilibrium of face ABCD

Taking moment about O

$$\tau_{xy} \times dy \times \frac{dx}{2} + \tau_{xy} \times dy \times \frac{dx}{2} - \tau_{yx} \times dx \times \frac{dy}{2} - \tau_{yx} \times dx \times \frac{dy}{2} = 0$$

$$\tau_{xy} \times dx\,dy = \tau_{yx} \times dx\,dy$$

$$\tau_{xy} = \tau_{yx}.$$

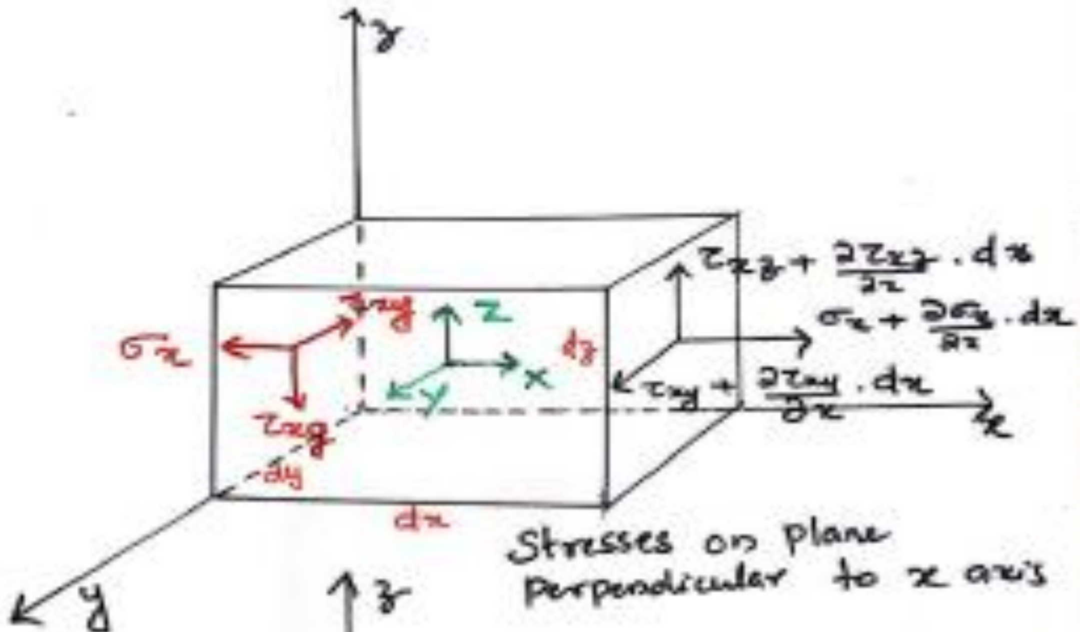
Similarly $\tau_{xz} = \tau_{zx}$

$\tau_{yz} = \tau_{zy}$

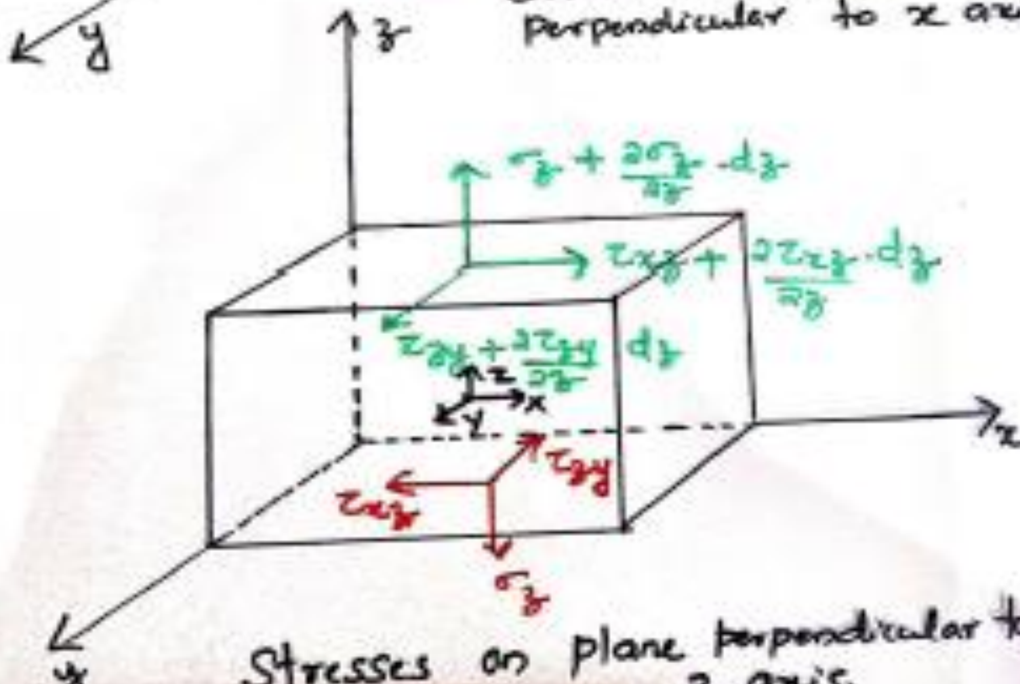
So stress tensor is symmetric, 9 components reduces to 6 components

$$\tau_{ij} = \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \end{array}$$

Equillibrium Equations in Cartesian Co-ordinates.



Stresses on plane perpendicular to x axis



Stresses on plane perpendicular to z axis.

$$\left.\begin{aligned}\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0\\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0\\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Z = 0\end{aligned}\right\} \quad \text{—— (1)}$$

Equilibrium equations in Cartesian co-ordinate system may be written in matrix form as

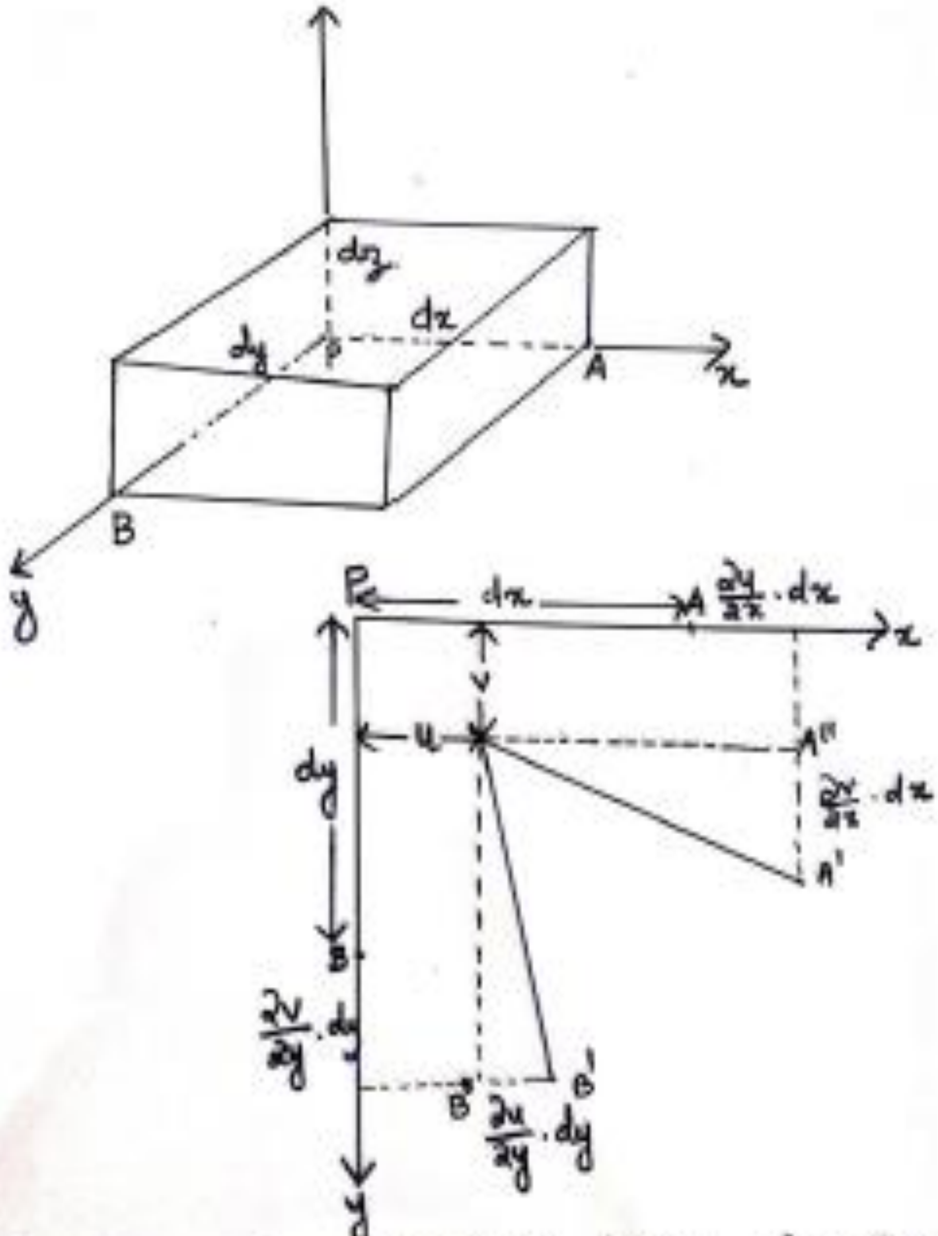
$$\underbrace{\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{bmatrix}}_{\text{Stress Tensor at a point.}} \underbrace{\begin{Bmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{Bmatrix}}_{\text{DIFFERCIAL OPEARATORS}} + \underbrace{\begin{Bmatrix}X\\ Y\\ Z\end{Bmatrix}}_{\text{BODY FORCE}} = \begin{Bmatrix}0\\ 0\\ 0\end{Bmatrix}$$

Strain and it's components

Strain is defined as the rate of relative displacement of two points with in a body. If the point are moving at equal distance in the same direction, such movement is called rigid body movement.

Fundamental Assumptions used in the derivation of strain components

1. The body is said to be strained when the relative positions of points in the body are altered.
2. There are enough constraints to prevent the body moving as a rigid body.
3. No displacement of a particle is posible without deforming a particle of the body.
4. Components of displacements u, v, and w varry continuously over the volume of the body



Consider an elementary block of sides dx, dy, d

The edges PA is along 'x' axis and PB is along 'y' axis.

Unit Elongation / Strain in x direction

$$= \frac{\frac{\partial u}{\partial x} \cdot dx}{dx}$$

ie $\varepsilon_x = \frac{\partial u}{\partial x}$

Simillarly $\varepsilon_y = \frac{\partial v}{\partial y}$

$$\varepsilon_z = \frac{\partial w}{\partial z}$$

Angle of distorsion of PA

$$= \frac{\frac{\partial v}{\partial x} \cdot dx}{dx} = \frac{\partial v}{\partial x}$$

Angular distorsion of PB $= \frac{\frac{\partial u}{\partial y} \cdot dy}{dy}$

$$= \frac{\partial u}{\partial y}$$

Total Angle of distorsion at P, γ_{xy}

$$= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

γ_{xy}, shear strain in XOY plane

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Simillarly other strains

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

Thus the state of strain at a point can be expressed as

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} \\ \varepsilon_z &= \frac{\partial w}{\partial z} \end{aligned}\right\} \text{Normal strain}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

The strain tensor in general form may be written as

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

where $i, j = x, y, z$

Strain tensor

Strain tensor at a point is given in matrix form as

$$\varepsilon = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

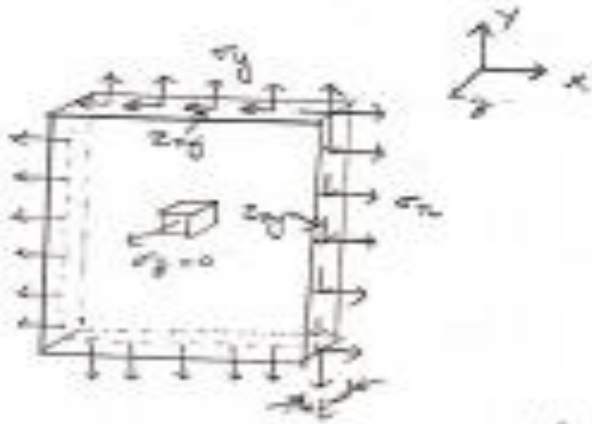
Strain tensor in terms of displacement can be written as

$$\varepsilon = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) & \frac{\partial w}{\partial z} \end{bmatrix}$$

Two dimensional problems in Theory of Elasticity

1) Plain stress case (stress components are independent of z dir)

If a thin plate is loaded by forces applied at the boundary parallel to the plane of the plate and distributed uniformly over the thickness as shown in figure



Since thickness is small, loading has no variation along the thickness. The stress components σ_z, τ_{xz} and τ_{yz} are zero on both phases of the plate and also it is assumed to be zero within the plate. Then the state of stress is called plane stress. Thus for plain stress

$\sigma_x, \sigma_y, \tau_{xy} = f(x, y)$

$$\tau_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The Governing equation of elasticity are

1) Equilibrium equations

$$\frac{\partial}{\partial x}\sigma_x + \frac{\partial}{\partial y}\tau_{xy} + X = 0$$

$$\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\sigma_y + Y = 0$$

$z = 0$.

2) Stress strain relations

$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \cancel{\sigma_z})]$

$= \frac{1}{E}[\sigma_x - \nu\sigma_y]$ —(A)

$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu\sigma_x]$ —(B)

$\varepsilon_z = \frac{-\nu}{E}(\sigma_x + \sigma_y)$ —(C)

here $\sigma_z = 0$
$\gamma_{xz} = 0$
$\gamma_{yz} = 0$
(since τ_{xz}, τ_{yz} are zeros).

$\gamma_{xy} = \frac{\tau_{xy}}{G}$

$G = \frac{E}{2(1+\nu)}$

$\gamma_{xy} = \frac{\tau_{xy}\,2(1+\nu)}{E}$ —(D)

3) Strain-displacement relations.

$\varepsilon_x = \frac{\partial u}{\partial x}$, $\varepsilon_y = \frac{\partial v}{\partial y}$, $\varepsilon_z = \frac{\partial w}{\partial z}$, $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

Example for plain stress case.

1) Deep beam with transverse loading
(depth and length >>> width of beam)

2) Masonry wall with transverse (vertical) loads.

From equation (A), B, (C) above.

$\varepsilon_x + \varepsilon_y = \frac{1}{E}[\sigma_x + \sigma_y - \nu(\sigma_x + \sigma_y)]$

$= \frac{(1-\nu)}{E}(\sigma_x + \sigma_y)$

$\sigma_x + \sigma_y = \frac{E}{(1-\nu)}(\varepsilon_x + \varepsilon_y)$.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix}\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Mohr's circle

is the graphical method for easily determining the normal and shear stresses without using the stress transformation equations.

Sign conventions

1) In cartesian co-ordinate axes, the horizontal axis represent normal stress on the plane and vertical axis represents shear stress.

2) Tensile stress is taken as +ve and marked rightwards from the origin, compressive stress is taken as −ve and marked leftwards from origin

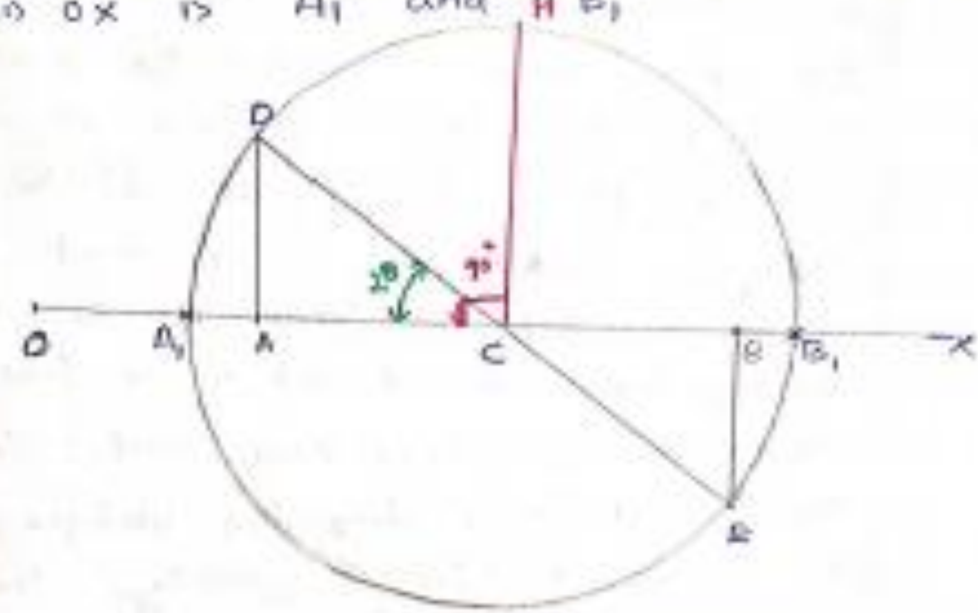
3) Clockwise shear stress is taken as +ve and anticlockwise shear stress taken as −ve.

This graphical method can be used to determine

a) The principal stress, maximum shear stress and state of stress along any oblique plane if the normal stresses and shear stress acting on the body are given.

b) normal, tangential and resultant stress on any oblique plane if the principal stresses acting on the body are given.

3) Now with C as center and radius CD or CE draw a circle. This is called Mohr's circle of stress. The point of intersection of the circle with axis OX is A_1 and B_1



Measure $OA_1 = \sigma_2$ (Minor principal stress)

$OB_1 = \sigma_1$ (Major principal stress)

So we have found out the principal stresses.

$\angle ACD = 2\theta$ (Measure $\angle ACD$) The principal stresses are inclined at angle θ and $90+\theta$ with the direction of σ_x.

4) Step-4

To find maximum shear stress draw a perpendicular CH from centre C. The line CH represents the maximum shear stress, τ_{max}.

$\angle ACH = 90$ (2θ) So $\theta = 45°$

This means τ_{max} is acting in a direction $45°$ inclined to the principal plane.

Mohr's Circle

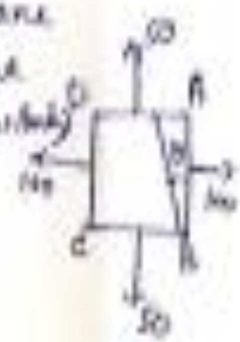
Example problems

Type-1 - When the state of stress at a point is given, $(\sigma_x, \sigma_y, \tau_{xy})$ and asked to find, principal stress, direction of principal plane, magnitude and direction of maximum shear stress plane.

1) A rectangular block of material is subjected to a tensile stress of $100\,N/mm^2$ on a plane and tensile stress of $50\,N/mm^2$ on a plane at right angles, together with a shear stresses of $60\,N/mm^2$ on the same planes. Find

1) The direction of the principal planes
2) The magnitude of the principal stresses.
3) The magnitude of the greatest shear stress.

Let 'θ' be the inclination of oblique plane (AB represents the plane carrying principal stress carrying normal stress of ...)



$$\tan 2\theta = \frac{2q}{\sigma_1 - \sigma_2} = \frac{2 \times 60}{100 - 50} = 2.4$$

$$2\theta = 67^\circ 23' \text{ or } 247^\circ 23'$$

$$\theta = 33^\circ 41' \text{ or } 123^\circ 41'$$

Major principal stress

$$\sigma_{1p} = \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+q^2}$$

$$= \frac{100+50}{2} + \sqrt{\left(\frac{100-50}{2}\right)^2+60^2}$$

$$\sigma_{1p} = 75 + 65 = 140\, N/mm^2 \text{ tensile}$$

$$\sigma_{2p} = 75-65 = 10\, N/mm^2 \, (T).$$

Maximum shear stress $q_{max} = \dfrac{\sigma_1-\sigma_2}{2}$

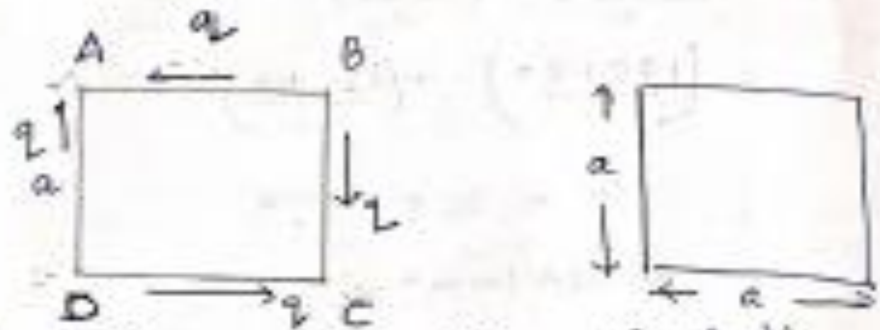
$$= \frac{140-10}{2} = 65\, N/mm^2$$

Maximum shear stress will occur on planes at $33^\circ 41' + 45^\circ = 78^\circ 41'$, and $78^\circ 41' + 90 = 168^\circ 41'$ with the plane AB carrying the normal stress of $100 N/mm^2$.

MODULE 2

Relationship between elastic constants

1) Relationship between Modulus of elasticity and Modulus of Rigidity (E and G)



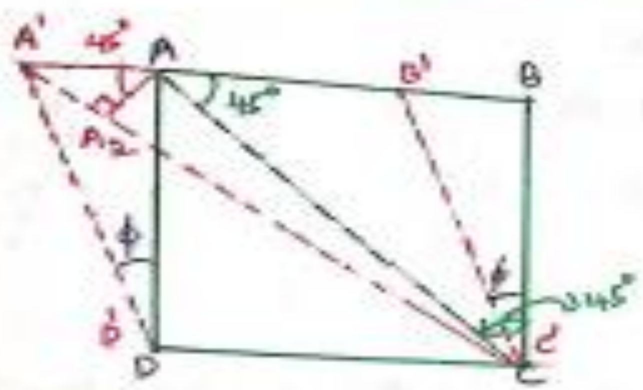
Consider a square block ABCD of side 'a' and thickness unity perpendicular to the plane of the drawing.

Let the block be subjected to shear stresses of intensity q as shown in figure.

Due to these stresses the be subjected to deformations such that diagonal AC is elongated and the diagonal BD is shortened.

In the case of pure shear, we have seen earlier than there is a diagonal tensile and diagonal compressive stress of equal intensity 'q' is induced.

The increase in length of the diagonal can be computed by considering the effect of diagonal tensile and diagonal compressive stress of intensity q.



ABCD original un-deformed shape of block

$AB'C'D'$ → deformed position of block.

'ϕ' angle of shear strain.

Increase in length of diagonal AC

= Strain in length of AC due to diagonal tensile stress on plane BD + strain in length of AC due to diagonal compressive stress on plane AC

$$\text{Strain of } AC = \frac{q}{E} + \frac{q}{mE}$$

$$= \frac{q}{E}\left(1+\frac{1}{m}\right) \quad \text{—— (A)}$$

Hence strain of diagonal

$$AC = \frac{q}{E}\left(1+\frac{1}{m}\right).$$

Strain in the length of AC can be determined by the geometry of the distorted shape of the block.

Increase in length of AC = $A'C - AC$

Let AA_2 be perpendicular to $A'C$. Hence $\angle ACA_2$ very small

$$AC = A_2C$$

$$AC = CA' - CA_2 = A'A_2 = AA' \cos \angle AA'A_2$$

$\angle BAC = 45°$, $\angle AA'A_2 = 45°$

$$A'A_2 = A'A \cos 45 = \frac{A'A}{\sqrt{2}}$$

Increase in length of diagonal AC $= \frac{AA'}{\sqrt{2}}$ — ①

Shear strain $\phi = \frac{A'A}{AD}$

$= \frac{A'A}{a}$

$A'A = a\phi \rightarrow$ substitute in ①

Increase in length of AC $= \frac{a\phi}{\sqrt{2}}$

length of diagonal AC $= a\sqrt{2}$

Strain of diagonal $= \frac{\text{Change in length}}{\text{Original length}}$

$= \frac{a\phi}{\sqrt{2}} \times \frac{1}{a\sqrt{2}}$

$= \frac{\phi}{2}$

From equation Ⓐ earlier

Strain of diagonal $= \frac{q}{E}\left(1+\frac{1}{m}\right)$

$\phi/2 = \frac{q}{E}\left(1+\frac{1}{m}\right)$

$\frac{q}{\phi} = \frac{\text{shear stress}}{\text{shear strain}} = G$, modulus of rigidity

$E = \frac{2q}{\phi}\left(1+\frac{1}{m}\right)$

$= 2G(1+\nu)$ $\quad \frac{1}{m} = \nu$ Poisson's ratio

Volumetric strain = 3 × change in diameter.

eg:-

1) A steel rod 4 meters long and 20mm diameter is subjected to an axial tensile load of 45kN. Find the change in length, diameter and volume of the rod. Take $E_s = 2 \times 10^5$ N/mm² and Poisson's ratio $= \frac{1}{4}$.

$$\text{Area} = \frac{\pi}{4} \times 20^2 = 314.2\,mm^2$$

$$\text{Tensile stress } p = \frac{\text{Load}}{\text{Area}} = \frac{45 \times 10^3}{314.2} = 143.2$$

$$\text{Strain of length} = \frac{p}{E} = \frac{143.2}{2 \times 10^5} = 0.0007$$

Increase in length = strain × original length

$= 0.000716 \times 4000$

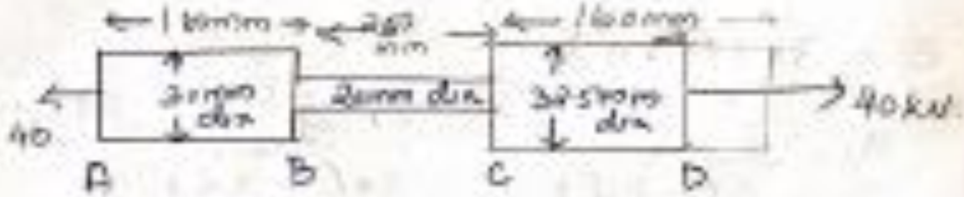
$= 2.864$ mm (+)

$$\frac{\text{Lateral strain}}{\text{Linear strain}} = \nu \text{ (Poisson's ratio)}$$

Lateral strain $= \frac{1}{4} \times 0.000716$ (−)

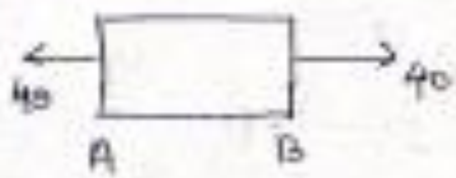
$= 0.000179$ −ve

1) Figure shows a bar consists of three lengths. Find the stresses in the three parts and the total extension of the bar for an axial pull of 40kN. Take $E = 2\times10^5$ N/mm².

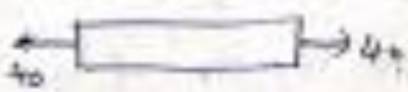


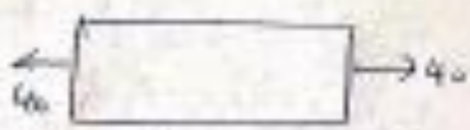
Intensity of stress in part AB

$$f_1 = \frac{P}{A_1} = \frac{40\times10^3}{\pi/4\times30^2} = 56.58\ \text{N/mm}^2$$



$$f_2 = \frac{P}{A_2} = \frac{40\times10^3}{\pi/4\times20^2} = 127\ \text{N/mm}^2$$





$$f_3 = P/A_3 = \frac{40\times10^3}{\pi/4\times32.5^2} = 48\ \text{N/mm}^2$$

.

Total extension

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{f_1}{E} \times l_1 + \frac{f_2}{E} \times l_2 + \frac{f_3}{E} \times l_3$$

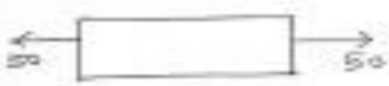
$$= \frac{1}{E}(f_1 l_1 + f_2 l_2 + f_3 l_3)$$

$$= \frac{1}{2\times10^5}(56.58 \times 160 + 127.3 \times 260 + 48 \times 16)\ mm$$

$$= 0.265\ mm$$

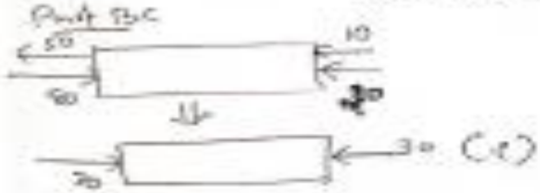
2) A member ABCD is subjected to axial forces as shown in figure. Find the total change in the length of the bar. Take $E = 1.05 \times 10^5$ N/mm², Area = 1000 mm².

Taking part AB



$$\delta l_1 = \frac{PL}{AE} = \frac{50 \times 600 \times 1000}{1000 \times 1.05 \times 10^5} = 0.2857\ mm\ (+ve)$$

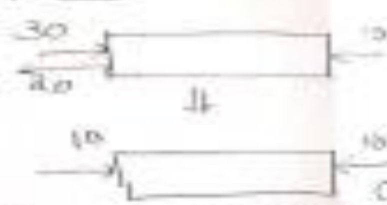
Part BC



contraction

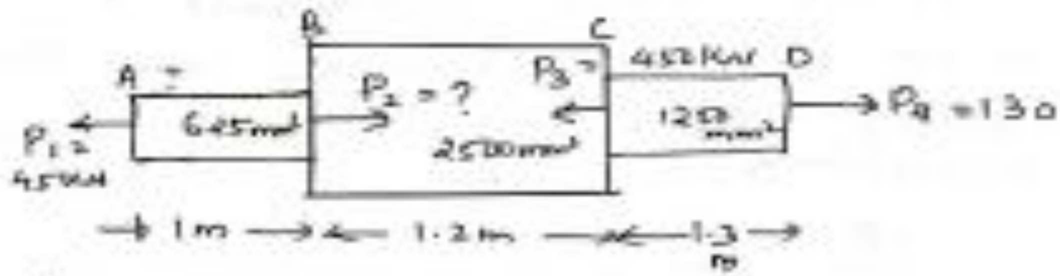
$$\delta l_2 = \frac{P_2 l_2}{AE} = \frac{30 \times 10^3 \times 1000}{1000 \times 1.05 \times 10^5} = 0.2857\ (-ve)$$

Part CD



$$\delta l_3 = \frac{10 \times 10^3 \times 1200}{1000 \times 1.05 \times 10^5} = -0.1143\ (c)$$

A member ABCD subjected to point loads P_1, P_2, P_3, P_4 as shown in figure. Calculate the force P_2, necessary for equilibrium if $P_1 = 45KN$, $P_3 = 450KN$ and $P_4 = 130KN$. Also determine the total extension of the bar $E = 2.1\times10^5$ Mpa.



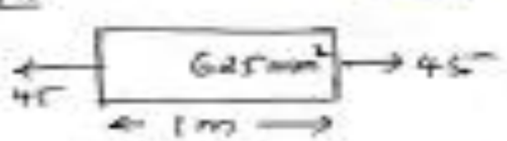
Ans: For equilibrium

$$P_1 + P_3 = P_2 + P_4$$

$$45 + 450 = P_2 + 130$$

$$P_2 = \underline{\underline{365KN}}$$

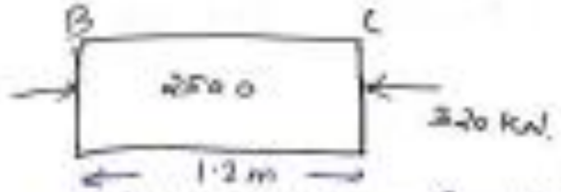
AB



+45kN

$$\delta l_1 = \frac{PL_1}{A_1E} = \frac{45\times10^3\times1000}{625\times2.1\times10^5}$$

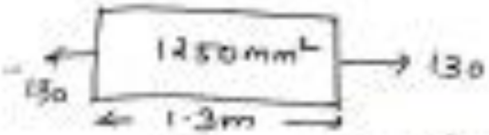
$$= 0.342mm \ (+)$$



Force on BC

+45 , −365 , net −320

$$\delta l_2 = \frac{320\times10^3\times1.2\times10^3}{2500\times2.1\times10^5} = \underline{0.7314mm}$$



Force on CD

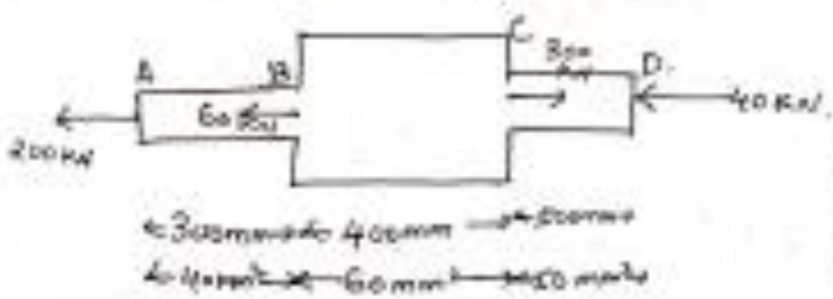
−320 , +450 net = +130

$$\delta l_3 = \frac{130\times10^3\times1.3\times1000}{1250\times2.1\times10^5}$$

$$= \underline{0.643mm}$$

$$\delta l = \delta l_1 - \delta l_2 + \delta l_3 = 0.25mm.$$

Q: Find the total elongation.

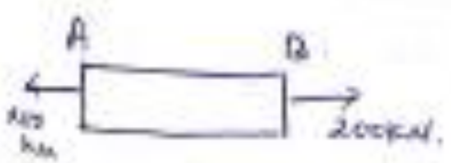


$E_{AB} = 2\times10^5 MPa$

$E_{BC} = 1\times10^5 MPa$

$E_{CD} = 2\times10^5 MPa$.

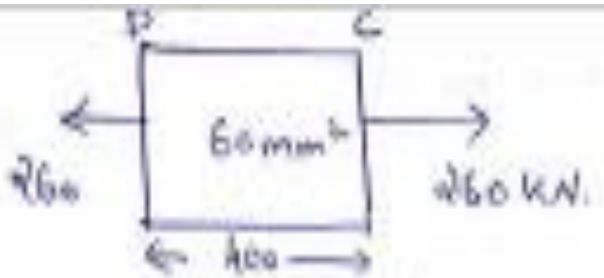
Ans



$$\delta l_1 = \frac{PL_1}{A_1E_1} = \frac{200\times10^3\times300}{40\times2\times10^5}$$

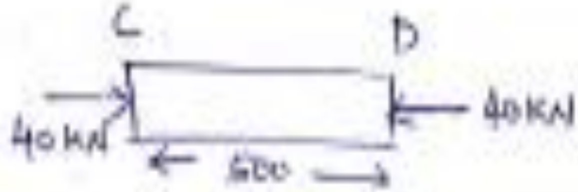
$$= \underline{7.5mm} \ (+ve)$$

· 200 +ve , 60 +ve



$$\Delta l_3 = \frac{P_2 L_2}{A_2 E_2} = \frac{260\times10^3 \times 400}{60 \times 1 \times 10^5}$$

$$= \underline{17.33\,mm} \ (+)$$



+260 (+ve) , and 200 ...

net −40

$$\Delta l_2 = \frac{40\times10^3 \times 500}{50 \times 2 \times 10^5}$$

$$= \underline{2\,mm} \ (-)$$

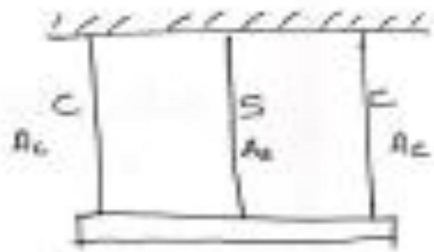
$$\Delta l = \Delta l_1 + \Delta l_2 - \Delta l_3$$

$$= 7.5 + 17.33 - 2$$

$$= 22.83\,mm$$

Extension of a tapering rod

Q) Three vertical wires of same length and in the same vertical plane together support a load of 20kN. The outer wires are of copper while the middle wire is of steel. The area of each wire is 100mm². The wires are so adjusted that each wire carries an equal load. An additional load of 20kN is now applied by a horizontal rigid bar. Find the stress in each wire. Find also what fraction of whole load is carried by the steel wire, Take $E_s = 2\times10^5\ N/mm^2$ and $E_c = 1\times10^5\ N/mm^2$



$A_s = A_c = 100\ mm^2$

Solution

Given each wire carries equal load

So load initially carried by each wire $= \frac{20}{3}$ kN.

Stress in each wire

$$p_s = p_c = \frac{20}{3}\ kN$$

$$= \frac{20\times10^3}{3\times100} = 66.67\ N/mm^2$$

Let f_s and f_c be the stresses due to an additional load of 20kN

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = \frac{E_s}{E_c}\times f_c = \frac{2\times10^5}{1\times10^5} f_c$$

$$f_s = 2 f_c$$

Load on steel + Load on copper (2 wires) = Additional load

$f_s \times 100 + f_c \times 2 \times 100 = 20 \times 10^3$

since $f_s = 2f_c$

$\Downarrow \quad 2f_c \times 100 + f_c \times 200 = 20 \times 10^3$

$$400 f_c = 20000$$

$$f_c = 50 N/mm^2$$

$$f_s = 50 \times 2 = 100 N/mm^2$$

Actual Stress in Copper = $p_c + f_c$

$= 66.67 + 50 = 116.67 N/mm^2$

Actual Stress in Steel = $p_s + f_s$

$= 66.67 + 100 = 166.67 N/mm^2$

Actual load on steel wire = Stress in Steel wire × Area

$= 166.67 \times A_s$

$= 166.67 \times 100 = \underline{16667 N}$

Fraction of load carried by Steel wire $= \dfrac{16667}{\text{Total load applied}}$

$= \dfrac{16667}{(20+20)10^3}$

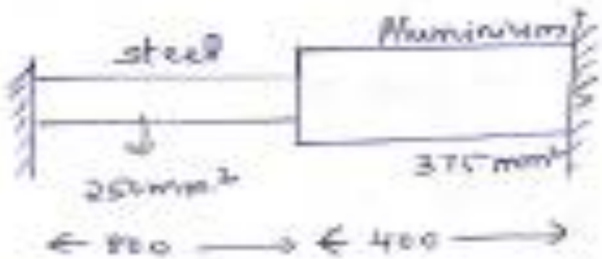
$= \underline{0.417}$

Q. The composite bar consists of steel and aluminium components as shown in figure is connected to two grips at the ends at a temperature of $60°C$. Find the stresses in the two rods when temperature falls to $20°C$. 1) if the ends do not yield 2) If the ends yield by 0.25 mm.

Take $E_s = 2\times10^5\ N/mm^2$, $E_a = 0.7\times10^5\ N/mm^2$

$\alpha_s = 1.17\times10^{-5}\ /°C$ $\quad \alpha_a = 2.34\times10^{-5}$ per °C.

Areas of the steel and aluminium bars are $250mm^2$ and $375mm^2$ respectively.



$$\frac{A_a}{A_s} = \frac{375}{250} = 1.5$$

Free contraction of the composite bar

$$\alpha_s T l_s + \alpha_a T l_a$$

$$= 1.17\times10^{-5}(60-20)\times800 + 2.34\times10^{-5}\times(60-20)\,400$$

$$= 0.7488\ mm$$

When the contraction is prevented completely or partially tensile stresses are induced in the rods. Let P_s and P_a be the stresses in the steel and aluminium rods.

Since the same force exists in the two rods

$A_s p_s = A_a p_a$

$$p_s = \frac{A_a}{A_s} p_a$$

$$= \frac{375}{250} p_a$$

$p_s = 1.5\, p_a$

Case (i) When the ends do not yield

Contraction prevented $= 0.7488$ mm

$$\frac{p_s}{E_s} \times l_s + \frac{p_a}{E_a} l_a = 0.7488$$

(Contraction Prevented in steel + Contraction Prevented in aluminium $= 0.7488$)

$$\frac{1.5\, p_a}{2 \times 10^5} \times 800 + \frac{p_a}{0.7 \times 10^5} \times 400 = 0.7488$$

$$11.7143 \times 10^{-3}\, p_a = 0.7488$$

$$p_a = 63.92 \text{ N/mm}^2$$

$$p_s = 1.5\, p_a = 1.5 \times 63.92 = 95.88 \text{ N/mm}^2$$

Case (ii) When the ends yield by 0.25 mm

Contraction Prevented $= 0.7488 - 0.25$

$= 0.4988$ mm

$$11.7143 \times 10^{-3}\, p_a = 0.4988 \text{ mm}$$

$$p_a = 42.52 \text{ N/mm}^2$$

$$p_s = 1.5 \times 42.52 = 63.87 \text{ N/mm}^2$$

[redacted] Linear elasticity – Generalised Hooke's law

The unique relationship between stress & strain is given by Hooke's law, Given by

$$\sigma_x = E\,\varepsilon_x \quad \text{(For uniaxial loading)}$$

where E = modulus of Elasticity.

Generalized Hooke's law is derived based on the principle that If more than one stress acts within elastic limits the stress developed at each point on the elastic body can be related to the respective strain components in a linear manner

$$\tau_{ij} = C_{ij}\,\varepsilon_{ij}$$

where τ_{ij} is the column matrix of stress components

ε_{ij} is the column matrix of strain components

C_{ij} is a 6×6 matrix of elastic constants

In general there are 81 elastic constants

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{yx} \\ \tau_{zy} \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{19} \\ C_{21} & C_{22} & \cdots & C_{29} \\ \vdots & & & \vdots \\ C_{91} & C_{92} & \cdots & C_{99} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{yx} \\ \gamma_{zy} \\ \gamma_{xz} \end{bmatrix}$$

stress vector = Constitutive matrix × Strain vector
(9 elements) 81 elements 9 elements

By applying $\sigma_{ij} = \sigma_{ji}$ and $\varepsilon_{ij} = \varepsilon_{ji}$ the 81 material constants reduces to 36

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & \cdots & & & C_{26} \\ - & - & - & & & - \\ - & - & - & - & - & \\ - & - & - & - & - & \\ C_{61} & - & C_{62} & C_{63} & \cdots & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

The constituent matrix C_{ij} is symmetric
the number of elastic constants reduces to 21

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix}$$

21 constants.

If U is the Strain energy
Strain energy is the energy stored in a body as a result of elastic deformation. The elastic work done to deform a member is equal to

the strain energy stored.

$$\text{Strain energy } U = \frac{\sigma^2}{2E} \times \text{volume} \quad , \bar{U} = \frac{\sigma^2}{2E} \text{ volume}$$

$$= \frac{\varepsilon^2 E^2}{2E} = \frac{\varepsilon^2 E}{2}$$

Partial derivative of strain energy w.r.t to strain $\frac{\partial \bar{U}}{\partial \varepsilon} = \frac{2\varepsilon E}{2} = \varepsilon E = \sigma$

caused by deformation $\frac{\partial \bar{U}_i}{\partial \varepsilon_i} = \sigma_i = \underbrace{C_{ij}}_{\text{constitutive matrix}} \cdot \varepsilon_j$

$$\frac{\partial^2 \bar{U}}{\partial \varepsilon_i \varepsilon_j} = C_{ij} \quad \text{—①}$$

$$\frac{\partial \bar{U}}{\partial \varepsilon_j} = C_{ji}\, \varepsilon_i$$

$$\frac{\partial^2 U}{\partial \varepsilon_j \varepsilon_i} = C_{ji} \quad \text{—②}$$

From ① and ② $C_{ij} = C_{ji}$

From ① and ②, we know that constitutional matrix is symmetric

Isotropic material

For a material whose elastic properties are not a function of direction at all, only two independent elastic material constants are sufficient to describe its behaviour completely. This material is called "isotropic linearly elastic" material.

Therefore the stress strain relationships may be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} 2G+\lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2G+\lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2G+\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

Therefore

$$\sigma_x = (2G+\lambda)\varepsilon_x + \lambda(\varepsilon_y + \varepsilon_z) \quad -①$$

$$\sigma_y = (2G+\lambda)\varepsilon_y + \lambda(\varepsilon_z + \varepsilon_x) \quad -②$$

$$\sigma_z = (2G+\lambda)\varepsilon_z + \lambda(\varepsilon_x + \varepsilon_y) \quad -③$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx}$$

where λ , Lame's constant

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad , \quad G = \frac{E}{2(1+\nu)}$$

adding ①, ②, ③

$$\sigma_x + \sigma_y + \sigma_z = (2G+\lambda)(\varepsilon_x + \varepsilon_y + \varepsilon_z) + 2(\varepsilon_x + \varepsilon_y + \varepsilon_z)\lambda$$
$$= (\varepsilon_x + \varepsilon_y + \varepsilon_z)(2G + 3\lambda)$$

MODULE 3

Beams and its types.

Beam- Beam is a structural member subjected to a system of external forces at right angle to its axis.

Types of beams

1) Cantilever beam.

One end fixed and other end is free

2) Simply supported beam.

Ends of the beam is freely resting on the supports.

3) Over hanging beam.

Either one or both of the ends of the beam projects beyond the support

4) Fixed beam

Both ends of beams are rigidly fixed

5) Continuous beam.

Beams having more than two supports

First three beams are statically determinate i.e, structure can be analysed using the equation of static equilibrium.

The last two are statically indeterminate.

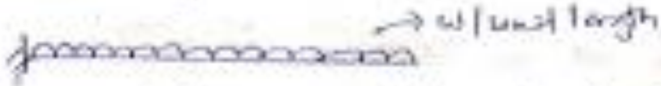
Types of loads

1) Point Load / concentrated Load

Loads are considered to be acting at a point.

W is considered to act at a point or a small distance.

5) UDL/Uniformly distributed load.



Uniformly distributed load is that whose magnitude remains uniform throughout the length.

3) Uniformly Varying Load.

It is the load whose magnitude is varying along the loading length with a constant rate.

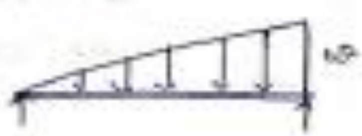
Uniformly Varying load is further divided into two groups.

1) Triangular Load

2) Trapezoidal Load.

Triangular Load

Triangular Load is that whose magnitude is zero at one end and and increases constantly till the other end or a portion of span

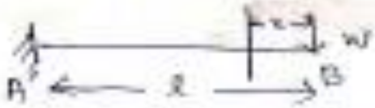


Trapezoidal load.

Trapezoidal load is that which is acting on the span length in the form of a trapezoid.

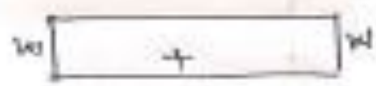
Case-a

1) Cantilever of length l carrying W at free end



Consider a section X at a distance x from free end

$SF_x = +W$ (Right downwards load +ve shear force)



Shear Force diagram

SFD

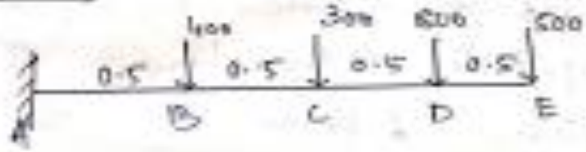
BM at $x = M_x = -W \cdot x$.

at $x = 0$, free end B $M_x = 0$

at $x = L$, fixed end A $M_x = -WL$

eg: Problems — Cantilever carrying several concentrated load.

1)



First take section between D and E

$SF_x = +500\ N$

$BM_x = -500 \times x$

at $x = 0$ ∴ at E $M_x = 0$

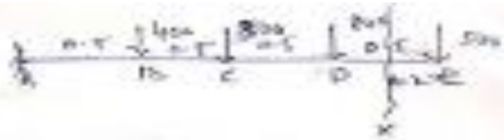
$SF = 500$

at $x = 0.5$, at D

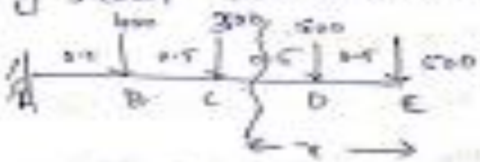
$SF = 500$

$M_x = -500 \times 0.5$

$= -250\,Nm$



at any section between C and D



$SF_x = +500 + 800 = 1300\,N$

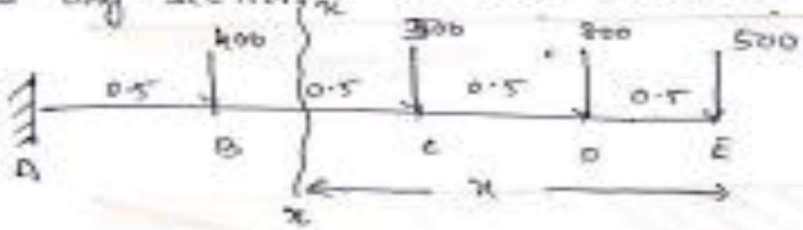
$BM_x = -500x - 800(x - 0.5)$

$= -1300x + 400$

at $x = 0.5$, at D $M_x = -1300 \times 0.5 + 400 = -250\,Nm$

at $x = 1$, i.e. at C $M_x = -1300 + 400 = -900\,N/m$

at any section x between B and C



$SF_x = 500 + 800 + 300 = 1600\,N$

$BM_x = -500x - 800(x - 0.5) - 300(x - 1)$

$= -1600x + 700\,Nm$

at $x = 1$ at C $M_x = -1600 \times 1 + 700 = -900\,Nm$

at $x = 1.5$ at B $M_x = -1600 \times 1.5 + 700$

$= -1700\,Nm$

At any section between A and B distant x from E

$SF = 500 + 800 + 300 + 400 = 2000 N$

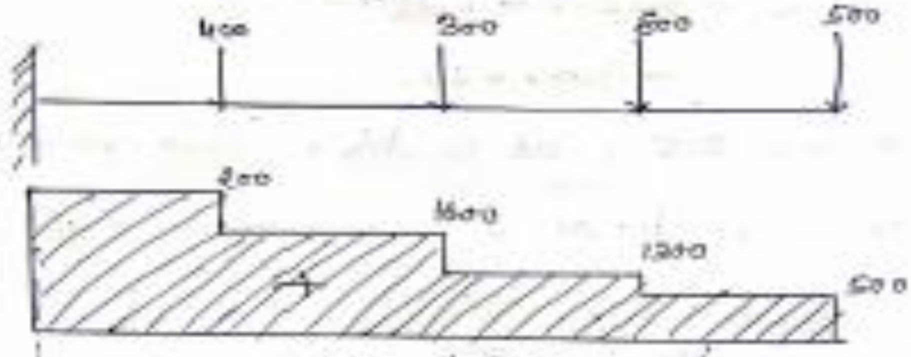
$BM = M_x = -500x - 800(x-0.5) - 300(x-1) - 400(x-1.5)$

$= -2000x + 1300 Nm$

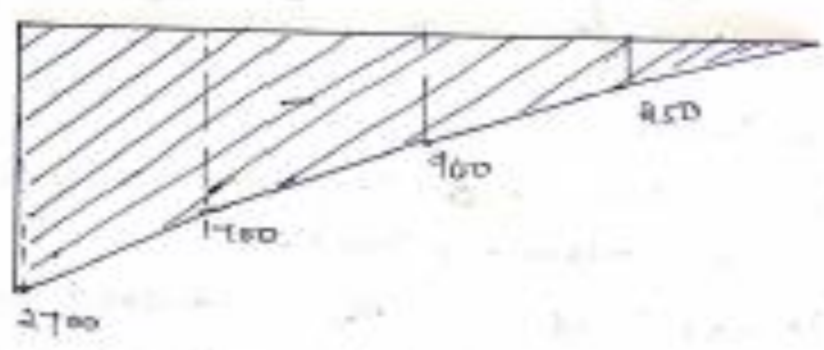
at $x = 1.5$, $M_x = -2000 \times 1.5 + 1300 = -1700 Nm$

at $x = 2$, $M_x = -2000 \times 2 + 1300$

$= \underline{-2700 Nm}$



SF diagram.



Cantilever subjected to a systems of loads.

Eg: Draw the shear force and bending moment diagram for the cantilever as shown in figure.

Let V_A be the vertical reaction at A. Since there only downward loads, V_A is acting upwards.

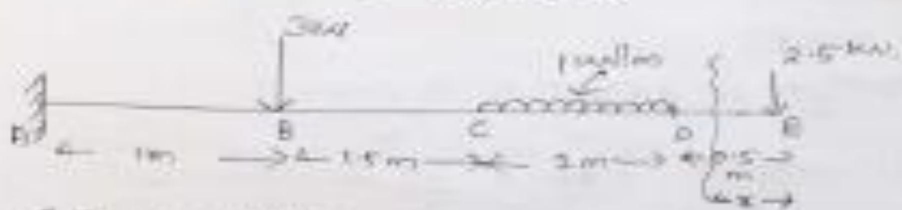
V_A = Total load on the span

$V_A = 3 + 2.5 + \underbrace{2\times1}_{(udl)} = 7.5\,kN$ (↑, upwards)

There will be also a reacting moment or fixing moment at A in an anticlockwise order, which will be equal and opposite to the moment of the forces on the cantilever about A.

Reacting moment $= 3\times1 + 2\times1\,(1+1.5+1) + 2.5\times5$

$= 22.5\,kNm$



S.F calculations

Take a section x between D and E

$SF_x = 2.5\,kN$

$SF_E = 2.5$, SF at D = 2.5 kN

Bending moment between D and E

$BM_x = -2.5x$ (x varies between 0 to 0.5)

$BM_{at E} = 0$ (x=0)

$BM_{at D} = -2.5 \times 0.5$ (x=0.5)

$= -1.25$ kNm

Section between C and D

Shear force

$SF_x = 2.5 + 1 \times (x - 0.5)$ (x b/n 0.5 to 2.5)

x=0.5 $SF_D = 2.5$

x=2.5 $SF_C = 2.5 + (2.5 - 0.5) \times 1$

$= \underline{4.5 \text{ kN}}$

Bending moment

$$BM_x = -\left(2.5x + 1 \times (x - 0.5)\frac{(x - 0.5)}{2}\right)$$

at $x = 2.5$

$$BM_C = -\left(2.5 \times 2.5 + \frac{(2.5 - 0.5)^2}{2}\right)$$

$$= -\left(6.25 + \frac{2^2}{2}\right)$$

$$= -\underline{8.25} \text{ kNm}$$

$BM_D = -2.5x$

$= -2.5 \times 0.5 = -1.25$ kNm.

at section between C and B

SF = 2.5 + 1×2

= 4.5 kN

SF at C = 4.5 kN

SF at B = 7.5 kN (3+4.5)

Bending moment

Bending moment at x $\quad BM_x = -\left(2.5x + 1\times2\times(x-(1+0.5))\right)$

$= -\left(2.5x + 2(x-1.5)\right)$ — ①

x varies b/n 2.5 (C) and 4 (2.5+1.5) (B)

BM at C (x = 2.5) from ①

$= -\left(2.5\times2.5 + 2(2.5-1.5)\right)$

$= -8.25$ kNm

BM at B (x = 4)

$BM_B = -\left(2.5\times4 + 1\times2\times(4-1.5)\right)$

$= -15$ kNm

Section between A and B

$SF_{at\,x} = 2.5 + 2\times1 + 3$

$= 7.5\ kN$

$SF_{at\,B} = 7.5\ kN$

$SF_{at\,A} = 7.5\ kN$

Bending moment at any section x (x b/w 4 to 5)

$= -\left(2.5x + 2\times1\times(x-1.5) + 3(x-4)\right)$

$BM_{at\,B}\ (x=4)$

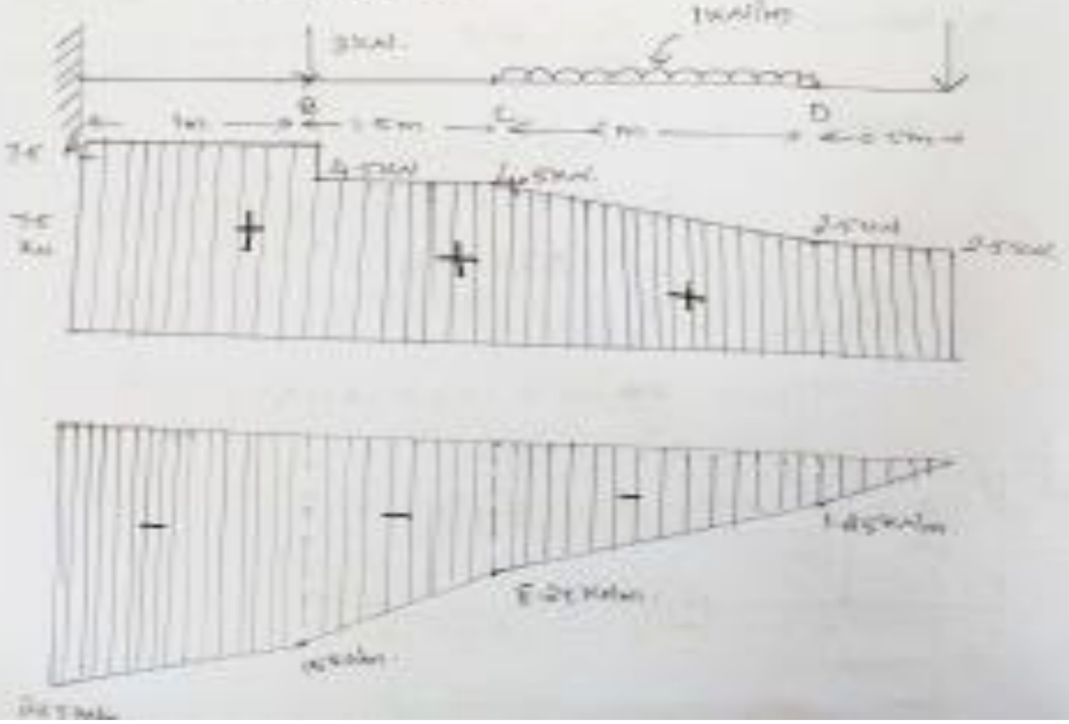
$= -\left(2.5\times4 + 2\times1(2.5) + 0\right)$

$= -15\ kNm$

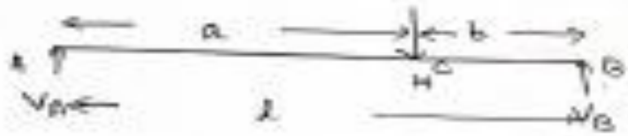
$BM_{at\,A}\ (x=5)$

$= -\left(2.5\times5 + 2\times1\times3.5 + 3(5-4)\right)$

$= -22.5\ kNm$



2) Simply supported beam with concentrated load placed eccentrically on the span.



$V_A + V_B = W$

Taking moment about A

$V_B \times l = W \times a$

$V_B = \frac{Wa}{l}$

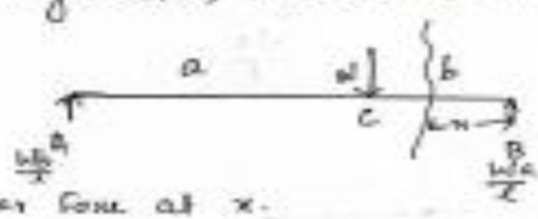
$V_A = W - \frac{Wa}{l}$

$= \frac{Wl - Wa}{l}$

$= \frac{W(a+b) - Wa}{l}$

$= \frac{Wb}{l}$

At any section between C and B



Shear force at x.

$SF_x = -\frac{Wa}{l}$

SF at C, at $x = b$

$SF_C = W - \frac{Wa}{l} \quad (x = b)$

$= +\frac{Wb}{l}$

At any section between C and B

$BM_x = +\frac{Wa}{l}x$ (sagging +ve)

at B (x = 0)

$BM_B = 0$

at C (x=b)

$BM_C = \frac{Wab}{l}$

At any Section between C and A.

$BM = +\frac{Wa}{l}x - W(x-b)$

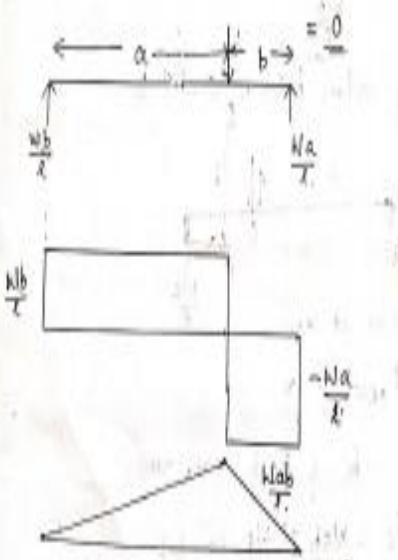
at x=b , $BM_C = \frac{Wab}{l} - W.0$

$= \frac{Wab}{l}$

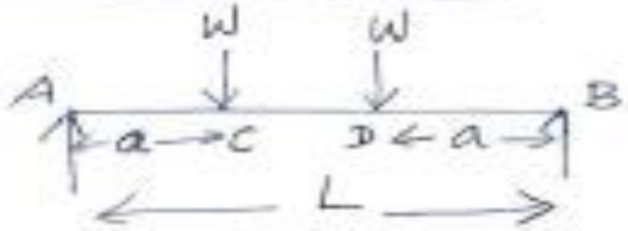
at $x = l = (a+b)$, $BM_A = \frac{Wa}{\cancel{l}} \times \cancel{l} - W(l-b)$

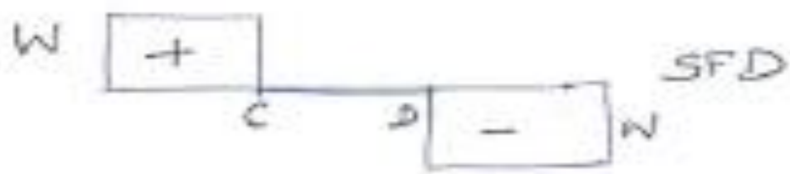
$= Wa - W(a+b-b)$

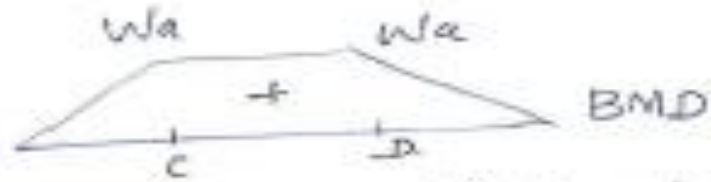
$= \underline{0}$



2020 Bending stress in Beams.







Consider a simply supported beam loaded as shown in figure. This is generally called two-point loading. In region CD, there is no shear force. It is subjected to constant BM which is equal to Wa. This region is said to undergo simple bending or pure bending.

Assumptions in theory of simple bending

1. The material of the beam is homogeneous and isotropic.
2. The transverse sections which were plane before bending remain plane even after bending.
3. The value of Young's Modulus (E) is the same in tension and compression.
4. The material obey's Hooke's law and it is stressed within its elastic limit.
5. Each layer of the beam is free to expand or contract, independently, of the layer above or below it.
6. The radius of curvature (R) is very large compared to the cross-sectional dimensions of the beam.

∴ Moment of resistance offered by the whole section $= \int \frac{E}{R} y^2 \delta a = \frac{E}{R} I$

where I – moment of inertia of the cross-section $= \int y^2 \delta a$

By equilibrium, this moment of resistance must be equal to the applied moment M.

$$\therefore M = \frac{E}{R} I$$

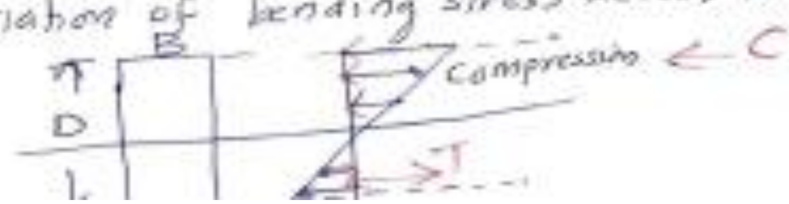
$$\text{or} \quad \frac{M}{I} = \frac{E}{R} \longrightarrow (2)$$

combining (1) and (2),

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$

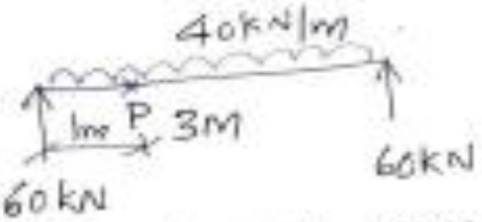
This is called equation of pure bending.

Using this equation, we can plot the variation of bending stress across the depth



1. A simply supported beam of length 3m carries a UDL of 40 kN/m over the entire span. It has a cross-section of 200mm × 400mm. Calculate the bending stress at a point 100mm above the bottom and 1m from the left support.

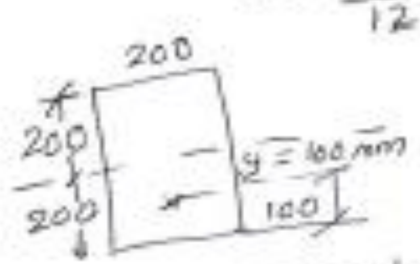
Step 1 Find out BM at the required section.



Bending moment at P $= 60 \times 1 - 40 \times 1 \times 0.5$

$M = 40\,kN\text{-}m$

$I = \frac{BD^3}{12} = \frac{200 \times 400^3}{12} = 1066.67 \times 10^6\ mm^4$



Bending equation is $\frac{M}{I} = \frac{f}{y}$

∴ Bending stress at the given point

$= \frac{M}{I} \times y = \frac{40 \times 10^6}{I} \times 100 = \underline{3.75\ N/mm^2}$

Torsion of circular shafts.

When a shaft is under pure torsion, its cross-sections are under pure shear stresses.

Equation of torsion

Assumptions:

1. Material of the shaft is homogeneous and isotropic.
2. Plane cross sections of the shaft remain plane and circular before and after twisting.
3. All diameters of the c/s of the shaft remain straight, with their lengths unchanged before and after twist.
4. The twist is uniform along the length of the shaft.
5. Stresses induced in the shaft due to torsion do not exceed the proportionality limit.
6. The relative rotation between any two cross-sections of the shaft is proportional to the distance between them.

Consider an elementary area dA on the cross-section at a radial distance (r) from the centre of the section.

Let τ be the shear stress induced in dA

Shear force (dF) acting on the element $= \tau \cdot dA$

Moment of this shear force about O,

$$dM = (\tau\, dA)\, r$$

From equation (3) $\frac{f_s}{R} = \frac{\tau}{r} = \frac{N\theta}{L}$

$$\therefore dM = \left(\left(\frac{N\theta}{L}\right) r\, dA\right) r = \frac{N\theta}{L} r^2 dA$$

$\therefore$ Total moment of resistance $= \frac{N\theta}{L} \int r^2 dA = \frac{N\theta}{L} J$

From equilibrium, this moment of resistance is equal to the applied torque (T)

$$T = \frac{N\theta}{L} J$$

J – polar moment of inertia $= \frac{\pi D^4}{32}$

$$\boxed{\frac{T}{J} = \frac{N\theta}{L}} \longrightarrow (4)$$

$\therefore$ The torsion equal for circular shaft is

$$T = f_s = N\theta$$

7. Find the diameter of the shaft required to transmit 160 kW at 250 rpm, if the maximum torque is not to exceed the mean torque by 35% with a maxi. permissible shear stress of 50 N/mm^2

Power = $P = 160 \times 10^3$ Nm/s $= 160 \times 10^6$ N-mm/s

$$P = \frac{2\pi N T_{mean}}{60} = 160 \times 10^6$$

$$\frac{2\pi \times 250}{60} \times T_{mean} = 160 \times 10^6$$

$$T_{mean} = 6.11 \times 10^6 \text{ N.mm}$$

$$T_{max} = 1.35\, T_{mean} = 8.25 \times 10^6 \text{ N.mm}$$

$$\frac{T_{max}}{J} = \frac{f_s}{R}$$

$$16 \times \frac{8.25 \times 10^6}{\pi D^4} = \frac{50}{D/2}$$

Solving $D = \underline{\underline{95}}$ mm

Two shafts of same material and same length are subjected to same torque. If the first shaft is of a solid circular section and the second is of hollow circular section, whose internal diameter is 3/4th external diameter and the maximum shear stress developed in each of them are same, compare the weights of the two shafts

$$\frac{\text{Weight of solid shaft}}{\text{weight of hollow shaft}} = \frac{\pi/4\, D^2 L \rho}{\frac{\pi}{4}\left(D_0^2 - D_i^2\right) L \rho}$$

$$= \frac{D^2}{D_0^2 - (0.75 D_0)^2} = \frac{D^2}{0.4375\, D_0^2}$$

It is required to find a relation between dia of solid shaft (D) and outer diameter (D_0) of hollow shaft.

Given data → maxl. shear stress is same

Solid shaft → $f_{s_1} = \frac{T}{J} R = \frac{32T}{\pi D^4} \times \frac{D}{2}$

$$f_{s_1} \propto \frac{1}{D^3} \longrightarrow (1)$$

Hollow shaft, $f_{s_2} = \frac{T \times 32}{\pi\left[D_o^4 - (0.75 D_o)^4\right]} \times \frac{D_o}{2}$

$$f_{s_2} \propto \frac{D_o}{D_o^4 - 0.316 D_o^4} = \frac{1}{0.6836 D_o^3}$$

$$f_{s_1} = f_{s_2}$$

$$\frac{1}{D^3} = \frac{1}{0.6836 D_o^3}$$

$$D_o^3 = \frac{D^3}{0.6836} = 1.4628 D^3$$

$$\boxed{D_o = 1.135 D}$$

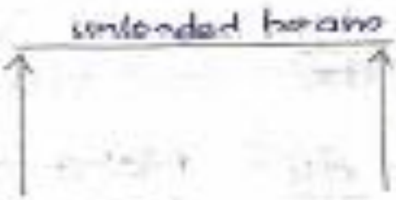
$\therefore$ Ratio of weights $= \frac{D^2}{0.4375 D_o^2} = \frac{1}{0.4375 \times 1.135^2}$

$= 1 : 0.5638$

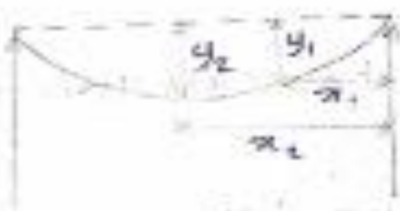
or $= \underline{\underline{1.77 : 1}}$

MODULE 4

DEFLECTION OF BEAMS



$\Rightarrow$ No deflection



$y \rightarrow$ Vertical deflection

$y_1 = x_1$

$y_2 = x_2$

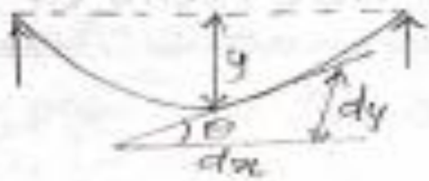
y Vs x

$\therefore \quad y = f(x)$

i.e. $\quad y = y_1, y_2, y_3 \ldots$

$x = x_1, x_2, x_3 \ldots$

Slope



$$\therefore \theta = \frac{dy}{dx}$$

$$\boxed{Slope = \theta = \frac{dy}{dx}}$$

Make a tangent of point. we get slope

Relationship b/w deflection and slope

$$\boxed{M = EI \frac{d^2y}{dx^2}}$$

E = Young's modulus

I = Polar moment of inertia

y = Deflection

$\frac{dy}{dx} = \theta =$ Slope

Analysing the deflection equation, we can use different methods.

Method 1:-

Double integration method

Method 2:-

Macaulay's method.

Method 3:-

Moment area method

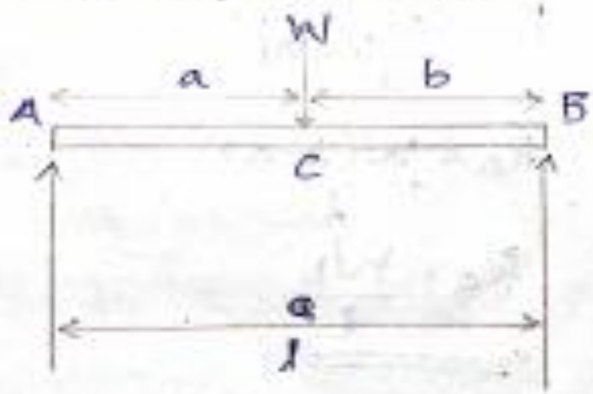
Method 4:-

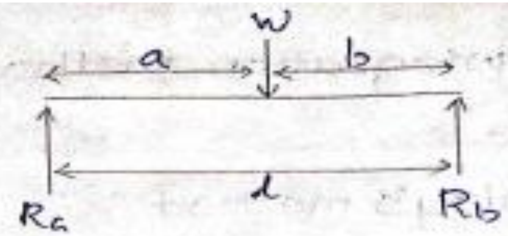
Conjugate beam method

Macaulay's method

Simply supported beam with a point load

Consider a simply supported beam of length l and carrying an eccentric point load W at point C which lies at distance 'a' from the left support 'A' and at distance 'b' from the right support 'B'





In static equilibrium of the beam $\Sigma F = 0$

$$R_a - W + R_b = 0$$

$$R_a + R_b = W$$

In Static equilibrium of the beam $\Sigma M = 0$

$$-R_b \times l + W \times a = 0$$

$$R_b l = W a$$

$$R_b = \frac{W a}{l}$$

$$R_a + R_b = W$$

$$R_a + \frac{W a}{l} = W$$

$$R_a = \frac{W(l - a)}{l}$$

$$R_a = \frac{W b}{l}$$

$$= \frac{Wbl^2}{6} + c_1 l - \frac{W(b)^3}{6}$$

$$c_1 = \frac{1}{l}\left[\frac{Wb^3}{6} - \frac{Wbl^2}{6}\right]$$

$$= \frac{1}{l}\left[\frac{Wb}{6}(b^2 - l^2)\right]$$

$$= -\frac{Wb}{6l}(l^2 - b^2)$$

Subotitute c_1 and c_2 values in eqn ① & ②

the equation for slope & deflection maybe wolttem as

$$\text{slope} = \frac{dy}{dx} = \frac{1}{EI}\left[\frac{Wb}{2l}x^2 - \frac{Wb}{6l}(l^2 - b^2) - \frac{W}{2}\frac{(x-a)^3}{3}\right]$$

$$\text{Deflection}: y = \frac{1}{EI}\left[\frac{Wb}{6l}x^3 - \frac{Wb}{6l}(l^2 - b^2)l - \frac{W}{6}(x-a)^3\right]$$

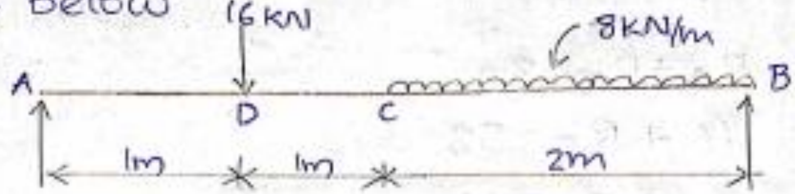
Slopes:-

Slopes is maximum at A or B The slope at A is workedout by putting $x=0$ in expoession

(iii) upto dotted line as point A lies in the position AC

$$\theta_a = \frac{1}{EI}\left[\frac{Wb}{2l} \times 0 - \frac{Wb}{6l}(l^2 - b^2)\right]$$

2 A simply supported beam of 4m span length is subjected to a concentrated point load and uniformly distributed length over right half of its span as shown below



Using Macaulay's method, make calculations for slope at A · deflection at centre of the beam and maximum deflection · The - fluxural rigidity for the given beam
$EI = 4000\ kNm^2$

Given,

$l = 4m$

$\theta_a = ?$

$y_c = ?$

$y_{max} = ?$

Fluxural rigidity, $EI = 4000\ kNm^2$

Static equilibrium conditions

$\Sigma F = 0\ ,\ \Sigma M = 0$

$\Sigma F =$
$R_B - 16 - 16 + R_A = 0$

Integrating twice

$$\cancel{EI\frac{dy}{dx} = \frac{16x^2}{2} - \frac{16(x-1)^2}{2}}$$

$$EI\frac{dy}{dx} = 16\frac{x^2}{2} + c_1 \Big| - \frac{16(x-1)^2}{2} \Big| - 4\frac{(x-2)^3}{3} \Big| \quad (1)$$

$$EIy = 8\frac{x^3}{3} + c_1x + c_2 \Big| - \frac{8(x-1)^3}{3} \Big| - \frac{1}{3}(x-2)^4 \Big| \quad (2)$$

Applying boundary conditions at A -

$x=0,\ y=0$

We consider only 1st term

~~Substituting~~

Sub $x=0,\ y=0$ in eqn (2)

$$EI\times 0 = 8\times\frac{0^3}{3} + C_1\times 0 + C_2$$

$$\therefore \underline{C_2 = 0}$$

At point B

Sub $x=4,\ y=0$ in eqn (2)

$$EI\times 0 = 8\times\frac{4^3}{3} + C_1\times 4 + 0 - \frac{8(4-1)^3}{3} - \frac{1}{3}(4-2)^4$$

$$C_1 = -23.33$$

Sub $c_1 = -23.33$ & $c_2 = 0$ in eqn ① & ②

$$\therefore \frac{dy}{dx} = \frac{1}{EI} \times 8x^2 - 23.33 \Big| - 8(x-1)^2 \Big| - \frac{1}{3}(x-2)^3 \Big|$$

$$y = \frac{1}{EI} \times \left(\frac{8}{3}x^3 - 23.33\right) \Big| - \frac{8}{3}(x-1)^3 \Big| - \frac{1}{3}(x-2)^4 \Big|$$

(I) slop at A where $x=0$

$$\frac{dy}{dx} = \theta_a = \frac{1}{4000}\left[8 \times 0 - 23.33\right]$$

$$\theta_a = -5.83 \times 10^{-3} \text{ radian}$$

(ii) diflection at C where $x=0$

($\because$ distance b/w A & C is 2m)

$$y_c = \frac{1}{4000}\left(\frac{8}{3} \times 2^3 - 23.33 \times 2 - \frac{8}{3}(2-1)^3\right.$$

$$y_c = -7 \times 10^{-3}$$

$$= -7\,mm \downarrow$$

Maximum deflection occurs at the point where slope is zero. It may be observed that the equivalent point load of $8 \times 2 = 16$ KN acts at 1m from the right support.

Elastic strain energy for axial loading transverse, spheres, bending

Consider load deflection diagram

Area under the curve = Strain energy = u

$$\boxed{u = \tfrac{1}{2} F \delta}$$

Strain energy is energy stored inside an elastic member when an external force is applied to the body

Question:-

1. How much strain energy stored during the application of

1. axial force
2. Shear force
3. bending moment
4. torque

$$U_1 = \int_0^s \frac{F_x^2\,ds}{2AE}$$

$$U_2 = \int_0^s \frac{F_y^2\,ds}{2AG}$$

$$U_3 = \int_0^s \frac{M_y^2\,ds}{2EI_z}$$

$$U_4 = \int_0^s \frac{T_x^2\,ds}{2GI_p}$$

∴ Total strain energy U =

$$U = U_1 + U_2 + U_3 + U_4$$

$$= \int_0^s \frac{F_x^2\,ds}{2AE} + \int_0^s \frac{F_y^2\,ds}{2AG} + \int_0^s \frac{M_y^2\,ds}{2EI_z} + \int_0^s \frac{T_x^2\,ds}{2GI_p}$$

$$= \frac{1}{2}\,\frac{T_x^2\,ds}{G\,I_p}$$

$$U = \frac{1}{2}\int_0^s \frac{T_x^2\,ds}{G\,I_p}$$

⟶ strain energy for entire length.

$$U_1 = \int_0^s \frac{F_x^2\,ds}{2AE}$$

$$U_2 = \int_0^s \frac{F_y^2\,ds}{2AG}$$

$$U_3 = \int_0^s \frac{M_y^2\,ds}{2EI_z}$$

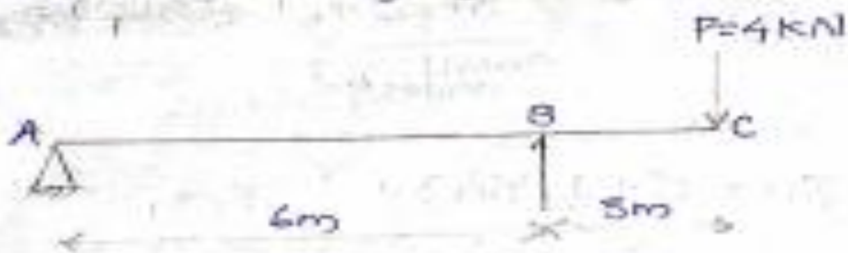
$$U_4 = \int_0^s \frac{T_x^2\,ds}{2GI_p}$$

∴ Total strain energy U =

$$U = U_1 + U_2 + U_3 + U_4$$

$$= \int_0^s \frac{F_x^2\,ds}{2AE} + \int_0^s \frac{F_y^2\,ds}{2AG} + \int_0^s \frac{M_y^2\,ds}{2EI_z} + \int_0^s \frac{T_x^2\,ds}{2GI_p}$$

Q. Use castigliano's method to determine the vertical deflection at the free end in the overchanging beam shawn below



$$R_A + R_B = P$$

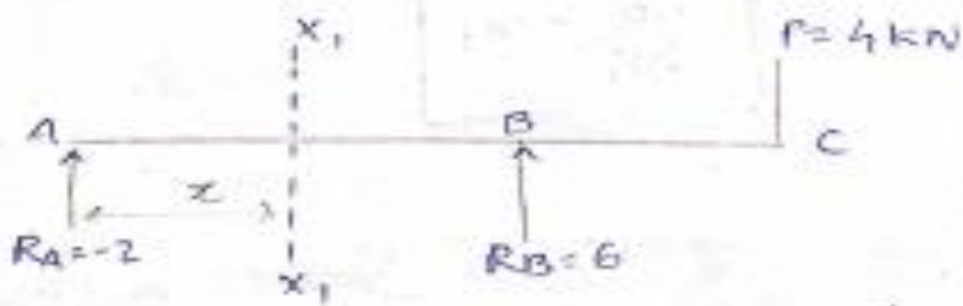
$$M_A = -6R_B + 36$$

$$R_B = 6$$

$$\therefore \quad R_A + R_B = 4$$

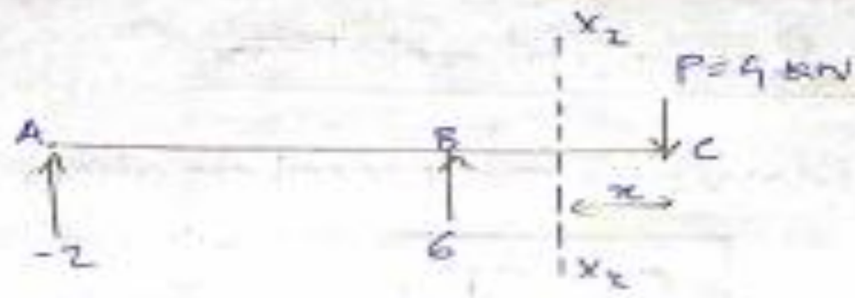
$$R_A + 6 = 4$$

$$R_A = -2$$



$$M_x = R_A \times x$$

$$= -2x$$



$M x_2 = Px$

$= 4x$

for segment AB moment $M' = -2x$ (Anti-clockwise)
and limit is from 0 to 6

for segmen BC moment $M' = 4x$ (clockwise)
limit is from 0 to 3

$\therefore$ strain energy $= \int \frac{M^2}{2EI} dx$

$$= \int_0^6 \frac{(-2x)^2}{2EI} dx + \int_0^3 \frac{(4x)^2}{2EI} dx$$

$$= \int_0^6 \frac{4x^2}{2EI} dx + \int_0^3 \frac{16x^2}{2EI} dx$$

$$= \frac{1}{2EI} \left[\left[\frac{4x^3}{3} \right]_0^6 + \left[\frac{16x^3}{3} \right]_0^3 \right]$$

$$= \frac{1}{2EI} \left[288 + 144 \right]$$

$$= \frac{1}{2EI} \times 432$$

$$= \frac{216}{2EI} = \frac{108}{EI}$$

Reciprocal relation

Maxwells reciprocal relation

$$\boxed{a_{ij} = a_{ji}}$$

Maxwells - Betti reciprocal relation

$$\delta_{21} \cdot F_2 = \delta_{12} \cdot F_1$$

Workdone by 1st system of forces due to displacement produced by 2nd system of force is equal to work done by the 2nd system of force due to displacement by 1st system of forces.

MODULE 5

BUCKLING OF COLUMNS, THEORIES OF FAILURE, THIN PRESSURE VESSELS

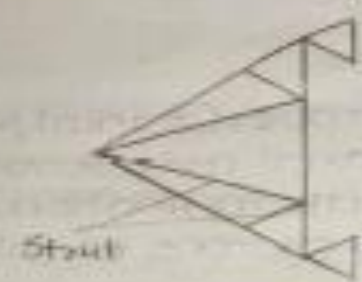
~~Columns and struts~~

Columns and Struts

Strut is a structural member (vertical/inclined) which is subjected to a axial compressive force. When strut is vertical with inclination 90° to the horizontal is called columns or pillars.

Strut	Column
* Shorter in length	* longer in length
* One or both ends of strut will be hinged or pin-joined.	* Both ends of column will be fixed
* Strut is horizontal, vertical or inclined.	* vertical member.
* Carry smaller compressive load.	* Vertical axial compressive load.
* Cross sectional dimension will be small.	* Cross sectional dimension will be large.
* Strut subjected to horizontal, vertical or inclined load.	* Line of action of compressive load pass through the axis of column or parallel to the axis of column.

Applications:-
roof trusses, Bridge-
- trusses



Buckling

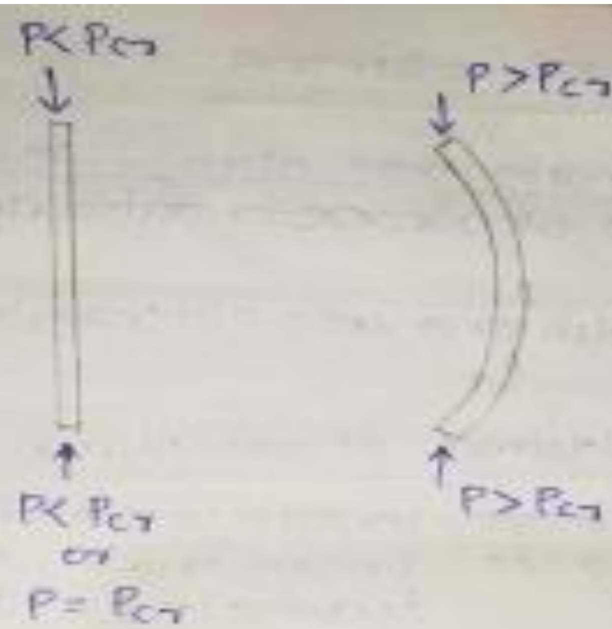
structural members such as columns that are long and slender subjected to an axial compressive force can deflect latera or sidewise if the force exceeds a critical value this is called

P = Axial compressive

P_{cr} = Critical load

Buckling occurs due to geometry / shape, boundary conditions and imperfections.

Critical load is the maximum axial load of a column can support (no buckling occurs)



Crushing and buckling load

The load at which failure of column occurs by crushing only is called crushing load. The short column fail due to crushing

The minimum axial load at which the column just buckles is called buckling load or crippling load

Slenderness ratio

It is the ratio of length of column to least radius of gyration.

Note:- **Failure of column**

Long column fails by buckling whereas short column fails by crushing

Case 1:-

Column with both ends pinned or hinged

Consider a column AB of uniform cross section hinged/pinned at both of its ends. When load P is applied the column bends

B.C

(i) At $x = 0$
$y = 0$

(ii) At $x = l$
$y = 0$

Applying these

at $x=0$, $y=0$

$$0 = C_1 \cos(m \times 0) + C_2 \sin(m \times 0)$$

$$0 = C_1 \times 1 + C_2 \times 0$$

$$\therefore C_1 = 0$$

at $x = l$, $y = 0$

$$0 = 0 \times \cos(m \times l) + C_2 \sin(m \times l)$$

$$C_2 \sin(ml) = 0$$

ie, $C_2 = 0$ or $\sin ml = 0$

$ml = 0$

$(\pi, 2\pi, 3\pi \ldots)$

Taking least values,

$$ml = \pi$$

$$\sqrt{\frac{P}{EI}}\, l = \pi \qquad \left(\because m^2 = \frac{P}{EI}\right)$$

Squaring both sides,

$$\frac{P}{EI} l^2 = \pi^2$$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

Proof

Rankine formula

$$\frac{1}{P_R} = \frac{1}{P_{cr}} + \frac{1}{P_E}$$

$$\frac{1}{P_R} = \frac{P_E + P_{cr}}{P_{cr} P_E} \Longrightarrow P_R = \frac{P_{cr} P_E}{P_E + P_{cr}}$$

$$P_R = \frac{P_{cr}}{\left(\frac{P_E + P_{cr}}{P_E}\right)} = \frac{P_{cr}}{\frac{P_E}{P_E} + \frac{P_{cr}}{P_E}} = \frac{P_{cr}}{1 + \frac{P_{cr}}{P_E}}$$

Sub $P_{cr} = \sigma_c A$ & $P_E = \frac{\pi^2 E I}{l^2}$

$$P_R = \frac{\sigma_c A}{1 + \frac{\sigma_c A}{(\pi^2 E I / l^2)}}$$

$$= \frac{\sigma_c A}{1 + \sigma_c A \times \frac{l^2}{\pi^2 E I}} \quad (\because I = Ak^2)$$

k = radius of gyrati

A - Area momen of inertia

$$= \frac{\sigma_c A}{1 + \frac{\sigma_c A}{\pi^2 E} \frac{l^2}{A k^2}}$$

$$= \frac{\sigma_c A}{1 + \alpha (l/k)^2}$$

Take $\alpha = \frac{\sigma_c}{\pi^2 E}$

$$\boxed{\alpha = \text{Rankine constant} = \frac{\sigma_c}{\pi^2 E}}$$

$$P_R = \frac{\sigma_c A}{1 + \alpha (l/k)^2}$$

Rankine formula for crippling load.

Find out length of column for which both Euler and Rankine formulae will give some values for crippling load.

i.e., $P_E = P_R$

$$\frac{\pi^2 EI}{l^2} = \frac{\sigma_c A}{1+\alpha\left(\frac{l}{k}\right)^2}$$

$$\frac{\pi^2 EI}{l^2}\left[1+\alpha\left(\frac{l}{k}\right)^2\right] = \sigma_c A$$

$$\pi^2 EI + \alpha\pi^2 EI\,(l/k)^2 = \sigma_c A l^2$$

$$\pi EI = \sigma_c A l^2 - \alpha\pi^2 EI\,(l/k)^2$$

We know $I = Ak^2$

$$\pi E k^2 = l^2\left(\sigma_c - \pi^2 E\alpha\right)$$

$$l^2 = \frac{\pi E k^2}{(\sigma_c - \pi^2 E\alpha)}$$

$$l = \left[\frac{\pi E k^2}{\sigma_c - \pi^2 E\alpha}\right]^{1/2}$$

modified form of Rankine formula is known as Ritter's formula

Critical load, $P = \dfrac{\sigma_c A}{1+\dfrac{\sigma_e}{\pi^2 E}(l/k)^2}$

σ_c = safe compressive stress

σ_e = elastic limit compressive stress

Q A 4m long hollow cast iron column of 300 mm external diameter and 225mm internal diameter has its both ends pinned (hinged) connection. Determine the safe compressive load the column can carry without buckling.
Make calculations using both the Euler's formula and Rankine formula and take

$E = 0.8 \times 10^5 \ N/mm^2$
Rankine constant $\alpha = \frac{1}{1600}$
crushing stress $= 600 \ N/mm^2$
factor of safety $= 2.5$

Given,

$l = 4m = 4 \times 10^3 mm$
$D_{ext} = 300 mm$
$D_{int} = 225 mm$
$E = 0.8 \times 10^5 \ N/mm^2$
$\alpha = \frac{1}{1600}$
crushing stress $= 600 \ N/mm^2$
Safe load $= ?$
Factor of safety $= 2.5$
Both ends hinged.

i) $P_E = \frac{\pi^2 E I}{l^2}$

$$P_E = \frac{\pi^2 E I}{l_e^2}$$

$$= \underline{134 \times 10^5 \text{ N}}$$

$$\text{Safe load} = \frac{P_E}{\text{factor of safety}}$$

$$= \frac{134 \times 10^5}{2.5}$$

$$= \underline{53.6 \times 10^5 \text{ N}}$$

b) using Rankine's formula

$$P_R = \frac{\sigma_c A}{1 + \alpha \left(\frac{l}{k}\right)^2}$$

$$A = \frac{\pi}{4}\left[D_{ext}^2 - D_{int}^2\right]$$

$$= \frac{\pi}{4}\left[300^2 - 225^2\right]$$

$$= \underline{30909 \text{ mm}^2}$$

$$k^2 = \frac{I}{A} = \frac{0.2718 \times 10^9}{30909} = \underline{8793.5}$$

$$P_R = \frac{600 \times 30909}{1 + \frac{1}{1600} \times \frac{(4 \times 1000)^2}{8793.5}}$$

$$= \underline{86.78 \times 10^5 \text{ N}}$$

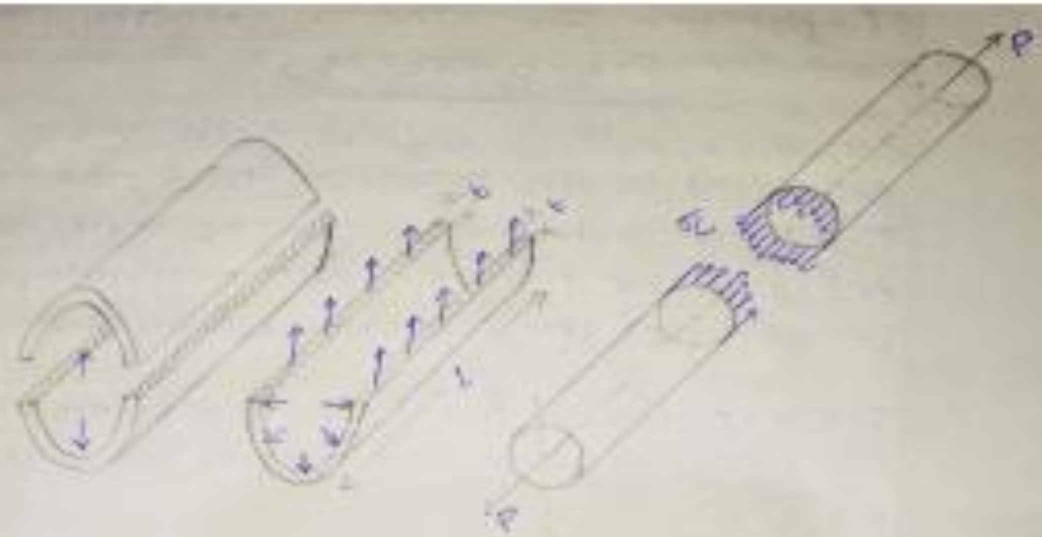
$$\text{Safe load} = \frac{86.78 \times 10^5}{2.5}$$

Longitudinal and circumferencial stress in a thin cylindrical vessel

* If the thickness of the cylinder is less than 20 times the internal of diameter then it is said to be thin cylinder otherwise it is thick cylinder
* used many engineering application
* Transporting or storing of liquids, gases or fluids

  eg:- Pipe, boiler, storage tank etc.
* These cylinders are subjected to internal fluid pressure
* stress distribution is assumed uniform over the thickness of the wall

Thin cylindrical pressure subjected to internal pressure



Why Bursting (failure) occur in thin cylindrical shell

Internal resistance offered by body - Should resist the tensile force offered by the fluid inside, the shell will safe

Otherwise if internal fluid pressure is more than resisting force due to - circumferential stress set up in the - material, the shell fail or bursting occur.

Limit case

Bursting force = Resisting force

Bursting force = Force due to internal fluid pressure

Resisting force = force due to circumferential or longitudinal stress

Q. A cylindrical shell 2.5m long which is closed at the ends has internal diameter 250mm and wall thickness 7.5mm.

Determine

(a) circumferential and longitudinal stresses induced in the shell material.

(b) change in length, diameter of the shell if it subjected to an internal pressure of $1.5\ MN/m^2$

take $E = 200\ GPa$ & poisons ratio $\mu = 0.3$

Given,

$L = 2.5\ m$

$d = 250\ mm = 0.25\ m$

$t = 7.5\ mm = 0.0075\ m$

$\sigma_c = ?$

$\sigma_L = ?$

$\Delta L = ?$

$\Delta d = ?$

$p = 1.5\ MN/m^2 = 1.5 \times 10^6\ N/m^2$

$E = 200\ GPa = 200 \times 10^9$

$\mu = 0.3$

Ans circumferential stress $\sigma_c = \frac{Pd}{2t}$

$= \underline{25 \times 10^6}\ N/m^2$

$$\sigma_L = \frac{Pd}{4t}$$

$$= \underline{\underline{12.5 \times 10^{6}\ N/m^2}}$$

(b) $$\varepsilon_c = \frac{\sigma_c}{E} - \mu \frac{\sigma_L}{E}$$

$$= 1.06 \times 10^{-4}$$

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu \frac{\sigma_c}{E}$$

$$= \frac{1}{E}\left[\sigma_L - \mu \sigma_c\right]$$

$$= 2.5 \times 10^{-5}$$

$$= 0.25 \times 10^{-4}$$

we know

$$\varepsilon_L = \frac{\delta l}{l}$$

$$\delta l = \varepsilon_l \times l$$

$$= 6.25 \times 10^{-5}$$

we know $$\varepsilon_c = \frac{\delta d}{d}$$

$$\delta d = \varepsilon_c \times d$$

$$= 2.65 \times 10^{-5}$$

$$\frac{\delta V}{V} = \varepsilon_L + 2\varepsilon_c$$

$$\delta V = V(\varepsilon_L + 2\varepsilon_c) \qquad \left| \; V = \frac{\pi}{4} d^2 l \right.$$

$$= 2.90 \times 10^{-5}$$

A thin cylindrical vessel subjected to internal fluid pressure and torque

The major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

Minor principal stress $= \dfrac{\sigma_1 + \sigma_2}{2} \sqrt{\left(\dfrac{\sigma_1 + \sigma_2}{2}\right)^2 + \tau^2}$

Maximum shear stress =

$$\frac{1}{2}(\text{Major principal stress} - \text{Minor principal stress})$$

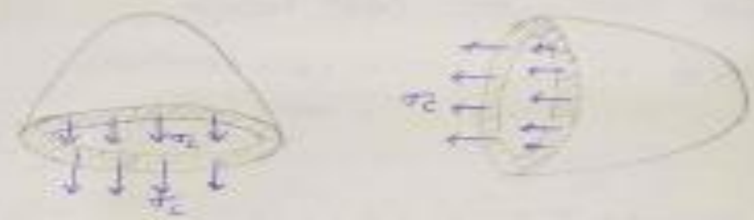
σ_1 = Circumferential stress ~~(tensile)~~ $= \sigma_c$

σ_2 = longitudinal stress $= \sigma_L$

τ = shear stress due to torque

Thin spherical shell

Consider a spherical shell of internal diameter 'd', thickness 't' subjected to internal pressure 'p'. Force due to internal pressure, the shell has a tendency to be torn away along the centre of sphere and split into 2 hemispheres.



Bursting force = Force due to internal pressure of fluid = $P \times$ Area of 'P' acting

$$= P \times \frac{\pi}{4} d^2 \text{ ——— } (1)$$

Resisting force = Force due to circumferential stress

$$= \sigma_c \times A$$

$$= \sigma_c \pi d t \text{ ——— } (2)$$

Limiting case

Bursting force = Resisting force

Equating (1) & (2)

$$P \times \frac{\pi}{4} d^2 = \sigma_c \pi d t$$

$$\sigma_c = \frac{Pd}{4t}$$

$\sigma_c = \sigma_1$

$\sigma_1 = \sigma_2$

$\sigma_1 = \sigma_2 = \frac{Pd}{4t}$

$\tau = \frac{\sigma_1 - \sigma_2}{2}$

$= 0$

Q A spherical shell of internal diameter 0.9 m and of thickness 10 mm is subjected to an internal pressure of $1.4\ N/mm^2$. Determine the increase in diameter and increase in volume. Take $E = 2\times10^5\ N/mm^2$ & $\mu = \frac{1}{3}$

Given: $d = 0.9\ m = 0.9\times10^3$

$t = 10\ mm$

$p = 1.4\ N/mm^2$

$E = 2\times10^5\ N/mm^2$

$\mu = \frac{1}{3}$

$$1-\mu = \frac{2}{3}$$

$$= \frac{1.4\times(0.9\times10^3)^2}{4\times10\times2\times10^5}\times\frac{2}{3}$$

$$= 0.0945$$

$$= \frac{\pi}{6}d^3 = \frac{\pi}{6}(0.9\times10^3)^3$$

$$= 3.817\times10^8$$

$$V = \frac{3\times1.4\times0.9\times10^3}{4\times10\times2\times10^5}\times\frac{2}{3}$$

$$= 1.2\times10^5$$

Theories of failure are:-

1) Maximum principle stress theory
(Rankine theory)

2) Maximum shear stress theory
(Coulomb Guest theory or
Tresk's theory)

3) Maximum principle strain theory
(St Venant theory)

4) Maximum strain energy theory
(Haigh's theory)

5) Maximum distortion energy theory
or shear strain energy theory
(Von-Misses & Henky's theory)

1) Maximum principle stress theory
(Rankine theory)

Rankine theory state that When a component subjected to bi axial or tri axial state of stresses, its failure occurs when maximum principle stress reaches the yield or ultimate strength of material

FOS

Region of safety

σ_1 & σ_2 are princip stress Biaxial state of stress can be represented as

The material will fail in any point having coordinates σ_1, σ_2 falls outside the square ABCD

this theory is applicable to brittle material

2 Maximum shear stress theory (coulomb Guest theory or Tresk's theory)

According to this theory failure of component which is subjected to biaxial or triaxial state of stresses occurs when the maximum shear stress at any point in the component equals the maximum shear stress reaches the yield ~~stress~~ point of material in a simple tension test.

The maximum value of shear ~~stress~~ in terms of principle stresses σ_1 & σ_2 is,

$$\cancel{\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2)}$$

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2)$$

or

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

3) Maximum principle strain theory

(St. Venant theory)

According to this theory, failure of material subjected to a stress occurs when the maximum principle ~~plane~~ strain reaches the strain at yield point of specimen subjecteda to simple tension test

According to generalised Hookes law $\varepsilon_1 = \frac{1}{E}(\sigma_1 - \mu(\sigma_2 + \sigma_3))$

consider a specimen subjected to uniaxial tension test have yield point strain is ε_1

$$\varepsilon_1 = \frac{\sigma}{E}$$

∴ in the limit

$$\frac{\sigma}{E} = \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right]$$

$$\left[\sigma = \sigma_1 - \mu(\sigma_2 + \sigma_3)\right]$$

4) Maximum strain energy theory

(Haigh's theory)

A material subjected to complex stresses fails when the total strain energy per unit volume at a point of machine component reaches the value of strain energy per unit value of the -

5) Maximum shear strain theory or maximum distortion theory

[Mises and Henkey's theory]

This theory states that the failure takes place when the shear strain energy in a complex system becomes equal to that in simple tension.

Shear strain energy in a complex system

$$\frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \text{ --- } (1)$$

G → modulus of rigidity

Shear strain energy in simple tension is found by inserting $\sigma_1=\sigma$, $\sigma_2=0$ and $\sigma_3=0$ in the above expression, ie,

shear strain energy in simple tension

$$=\frac{1}{12G}\left[(\sigma-0)^2+(0-0)^2+(0-\sigma)^2\right]$$

$$=\frac{2\sigma^2}{12G}=\frac{\sigma^2}{6G} \text{ --- } (2)$$

Therefore, in the limit (equating (1) & (2))

$$\boxed{\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]=2\sigma^2}$$

For a two dimensional stress system, the above relation may be reduced to

$$\left[(\sigma_1-\sigma_2)^2+(\sigma_2)^2+(-\sigma_1)^2\right]=2\sigma^2$$

$$\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2=\sigma^2$$